Welfare economics with status quo bias: a policy paralysis problem and cure

June 2004
(revised March 2006)

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Abstract:

To analyze welfare economics with status quo bias, we identify each agent with the incomplete preference relation defined by the preference judgments that hold at all of the agent's status quo points. Although the welfare theorems of general equilibrium theory continue to hold, the set of Pareto optima can be very large. For generic economies, almost every Pareto optimum sits amid an open set of Pareto optima, and the remaining measure-zero set of optima are on the boundary of this set. Thus, a small distortion would call for no policy response from a policymaker aiming for Pareto optimality. But these problems are specific to Pareto optimality as a welfare criterion. When a utilitarian planner faces agents with incomplete preferences, there will be a unique or at worst a low-dimensional set of optima. Moreover the utilitarian case for redistribution from low-marginal-utility agents to high-marginal-utility agents can be recast to cover agents with incomplete preferences. We also give several topological and measure-theoretic tests for whether or not an agent's preferences are substantially incomplete, and then show that if agents display status quo bias they pass all but one of these tests.

JEL Numbers: D11, D60, C60
Keywords: status quo bias, incomplete preferences, Pareto optimality, utilitarianism

1. Introduction

Traditional models of status quo bias or the endowment effect, e.g., Tversky and Kahneman (1991), posit a separate preference relation for each bundle of goods an agent might be endowed with. Unfortunately most of these preference judgments are unobservable. If an agent endowed with bundle a has the preferences $\succeq_a$, we can determine how $\succeq_a$ ranks a vis-à-vis an alternative bundle by seeing if the agent when endowed with a agrees to switch to the alternative, but we will not be able to see how $\succeq_a$ ranks among all the non-a alternatives. If when endowed with a the agent agrees to switch to either b or c we will not know whether $\succeq_a$ ranks b above c or vice versa, and similarly when the agent refuses to switch to either b or c. With such limited access to preferences, welfare analysis would be severely handicapped. We could of course fill in the missing preference ranking for b versus c by moving the agent's endowment to b or c and seeing if the agent agrees to switch to the other bundle. But with status quo bias the agent might well stick to b when endowed with b and stick to c when endowed with c. If we cannot judge a single individual's well-being consistently, *a fortiori* we cannot judge society's welfare.

Our attack on this problem is to extract the preference judgments that each agent makes unambiguously, that hold regardless of the agent's endowment point. If an agent endowed with a gives up a in favor of b, let us label this behavior as $b \succeq a$. By considering each endowment in turn and taking the union of all the binary preferences observed, we can unify the preference judgments the agent makes at different endowment points. This single preference relation can then serve as the data for welfare analysis. If the agent displays status quo bias, this ordering will not be complete: an agent endowed with a might not select b, and when endowed with b might not select a. The welfare economics of status quo bias thus ends up as the welfare economics of incomplete preferences. In the interpretation we give to $\succeq$, an agent indifferent between a and b will exchange one bundle for the other, but we could instead suppose that an agent will give up the status quo only for a strictly preferred bundle; we would then take strict rather than weak preference as primitive (see section 2).

When status quo bias is systematic, preferences will exhibit considerable incompleteness. ‘Systematic’ will mean that a willingness-to-accept/willingness-to-display (WTA/WTP) disparity obtains in any direction of movement away from an agent’s reference consumption bundle a: the boundary of the set of bundles preferred to a displays a kink at a no matter what direction in the commodity space we move. Mathematically this means that each agent’s ‘normal cone’ – the set of directions perpendicular to the budget planes that support agents’ sets of preferred bundles – has maximal dimension. We will show that this form of status quo bias implies that a full-dimensional set of bundles will be unranked relative to a (that is, no preference between a and these bundles obtains).

The preferences that arise from status quo bias can satisfy all of the other classical assumptions of economic analysis besides completeness. They can be transitive, allowing welfare inferences to be internally consistent. And they can be locally nonsatiated, continuous, and convex, thus allowing the two welfare theorems to apply: any competitive equilibrium is Pareto optimal and any Pareto optimum is a competitive equilibrium for an appropriate set of transfers. But these results do not tell us how discriminating Pareto optimality is as a welfare criterion. We will see that if agents exhibit the systematic status quo bias, then for generic models almost every optimum lies amid an open set (hence a set of maximal dimension) of Pareto optima. There can be boundary Pareto optima but they form a measure zero set and sit on the edge of the fully indeterminate optima. At the typical optima, therefore, a severe policy paralysis sets in. For example, suppose an economy’s initial allocation is one of the typical optima and the model is perturbed slightly, say by the addition of a small tax on net trades (with revenue redistributed to agents). Then the initial allocation remains optimal: even a paradigmatic tax distortion calls for no policy response.

We begin by assuming that agents experience status quo bias (a WTA-WTP disparity) along any direction of movement from any given reference bundle a, which implies that each agent judges a full-dimensional subset of bundles to be unranked relative to a. But we could instead suppose that status quo bias and hence incompleteness applies to some subset of the economy's goods. We then show that the set of Pareto optimal allocations of *these* goods then has maximal dimension.

But the news is not all bad. While status quo bias undercuts the discriminating power of Pareto optimality, other welfare criteria can still identify a small set of optimal allocations. Specifically we show that a utilitarian planner facing agents with incomplete preferences will either select a unique optimum or at worst choose from a low-dimensional set of optima. While a classical utilitarian planner facing complete preferences can do better and always declare a unique optimum, utilitarianism with incomplete preferences at least does not lead to the extreme dimensional expansion entailed by Pareto optimality. Utilitarianism thus "cures" the paralysis that occurs with Pareto efficiency. The basic cardinality building blocks that a utilitarian planner uses to compare utility across agents are no more demanding a construction than they are with complete preferences. Incompleteness moreover does not undercut the core utilitarian prescription that goods should be redistributed from low to high marginal utility agents. But with incomplete preferences this formula applies good by good: the planner makes a separate evaluation for each good k and individual i of how heavily to weight i's cardinal utility for k and then redistributes k accordingly. When preferences are complete, a utilitarian planner makes just one weighting per individual.

Our main purpose is to link status quo bias to incompleteness and then consider the implications for the number of Pareto and utilitarian optima. But not all varieties of preference incompleteness influence the size of the Pareto set; there are some trivial exceptions. For instance consider an agent with complete and transitive preferences $\succeq$ on bundles in R_+^L such that the resulting strict preferences $\succ$ are strictly convex, and for some bundle a relabel all of the bundles b indifferent to a as unranked. That is, for any b such that $b \sim a$ eliminate both (a, b) and (b, a) from $\succeq$. Perform such excisions for several or even every $a \in R_+^L$ thereby defining a new "relabeled" preference relation $\succeq_i^{INC}$. Now consider a society of I such agents, where for each agent i we specify an original preference relation $\succeq_i$ leading to the economy E and a relabeled preference relation $\succeq_i^{INC}$ leading to the economy E^{INC}. Then the sets of Pareto optimal allocations for E and E^{INC} must coincide. Since each $\succeq_i^{INC}$ is a subset of the original $\succeq_i$ it must be that if the allocation $x = (x_1, ..., x_I)$ Pareto dominates $y = (y_1, ..., y_I)$ in E^{INC} then x Pareto dominates y in E. Hence if z is Pareto optimal in E then z must also be Pareto optimal in E^{INC}. To see that the reverse implication holds, suppose y is Pareto optimal in E^{INC} but not in E. Then some x Pareto dominates y in E but not in E^{INC}. Since however the $\succeq_i^{INC}$ omit only indifferences from the $\succeq_i$ there must then be some i with $y_i \sim_i x_i$ in E. But since each $\succ_i$ is strictly convex, some convex combination of x and y would then Pareto dominate x in E^{INC}.

Since such trivial forms of incompleteness have no implications for the Pareto set, our first task, in the next section, will be to consider various substantial forms of incompleteness. We will see that status quo bias entails several strong forms of incompleteness. In the following section, we turn to the dimension of the Pareto set under status quo bias. Our final two topics are the dimension of the Pareto set when agents display status quo bias for a subset of goods and the number of utilitarian optima.

While the dimension of the Pareto set in incomplete-preference economies and the comparative efficacy of utilitarianism are novel topics, several of our points follow in the footsteps of Rigotti and Shannon (2005). They establish indeterminacy of the Pareto set in economies with uncertainty where incomplete preferences are represented as a set of probability distributions (Dana (2004) shows indeterminacy of the Pareto set for a specific case of incomplete preferences). Our characterization of Pareto optimal allocations as those where agents' normal cones have a direction of mutual intersection also parallels the treatment in Rigotti and Shannon. Billot et al. (2000) use a similar construction.

Our setting is distinctive in a couple respects. First incompleteness of preference does not by itself lead to nontrivial normal cones (as the example of trivial incompleteness above indicates). Instead it is a consequence of status quo bias. Second we must allow agents' normal cones to change as their consumption changes, which leads the manifold of consumption-normal vector pairs to be our primitive description of agents. The dependence of normal cones on consumption leads to the principal mathematical wrinkle we are forced to confront: boundary Pareto optima inevitably arise when agents' normal cones 'just' overlap, that is, when normal cones share a common direction but their interiors do not. For more on the normal cone characterization of Pareto optimality in this more general setting where normal cones are a function of consumption, see Bonnisseau and Cornet (1988).

At a technical level, a prime purpose of this paper is to marry the best-documented phenomena of behavioral economics – status quo bias – to the mathematical tools of general equilibrium theory and with the 'regularity' literature in particular. We will be able to characterize when agents with status quo bias combine together to create mathematically tractable Pareto sets and show that this situation is generic. Since the "better than" sets we consider are kinked, it is a pleasant surprise that the smooth tools of differential topology provide handy descriptions of both the well-behaved and troublesome configurations of incomplete-preference agents. Smooth analyses of the production sets that arise from activities, which are also kinked, offer a precedent (see Mas-Colell (1985)) and suggested the present approach.

For more on the connection between incomplete preferences and status quo bias, see Mandler (2004a) and Masatlioglu and Ok (2005), and earlier Bewley (1986). When we consider utilitarianism in section 5, we will need utility representations of incomplete preferences; here we follow the sets-of-utility-functions line of research (Ok (2002)) rather than the interval order tradition (Manzini and Mariotti (2004)).

2. Status quo bias and incompleteness

An individual is described by a preference relation $\succeq$ defined on R_+^L where L is the number of goods. Let $\mathscr{L}$ denote $\{1, ..., L\}$. The asymmetric part of $\succeq$ is denoted $\succ$, the symmetric part is denoted $\sim$, and the complement of $\succ$ is denoted $\nsucc$. The preferences $\succeq$ are *reflexive* if and only if $a \succeq a$ for all $a \in R_+^L$. We use two different transitivity conditions: $\succeq$ is *transitive* if and only if for all $a, b, c \in R_+^L$, $a \succeq b$ and $b \succeq c$ imply $a \succeq c$, and $\succeq$ is *weakly transitive* if and only if for all $a, b, c \in R_+^L$, $a \succeq b$ and $b \succeq c$ imply $c \nsucc a$. Weak transitivity ensures that the agent cannot be money-pumped or otherwise manipulated into endowment diminishing trades (see Mandler (2005)). When $\succeq$ is complete, weak transitivity and standard transitivity are equivalent. The relation $\succeq$ is: *convex* if and only if for all $a \in R_+^L$ the sets $B(a) \equiv \{b \in R_+^L : b \succeq a\}$ and $\{b \in R_+^L : b \succ a\}$ are convex, *strictly convex* if and only if, for all $a, b \in R_+^L$ and $\lambda \in (0, 1)$ $a \succeq b$ implies $\lambda a + (1 - \lambda)b \succ b$, *locally nonsatiated* if and only if, for all $a \in R_+^L$ and $\varepsilon > 0$, there is some $b \succ a$ with $\|a - b\| < \varepsilon$, *monotone* if and only if, for all $a \in R_+^L$ and $b \in R_{++}^L$, $a \succeq a + b$. We also define, for any $a \in R_+^L$, $W(a) \equiv \{b \in R_+^L : a \succeq b\}$. Finally, $\succeq_i$ is *locally nonindifferent* if $a \succeq b$ implies that for all $\varepsilon > 0$ there exists a $c \in R_+^L$ such that $c \succ b$ and $\|c - a\| < 0$.

Our primitive is the weak preference relation $\succeq$. As explained in the introduction, we take $a \succeq b$ to mean that an agent endowed with b accepts a in exchange. Incompleteness of $\succeq$ therefore occurs if status quo bias or an endowment effect is present, e.g., if a is not selected when b is the endowment and b is not selected when a is the endowment. Momentarily we give more structure to status quo bias, to match the behavioral evidence more closely.

If agents refuse to switch from status quo bundles unless offered *strictly* preferred alternatives, we could instead take $\succ$ as primitive. But if we begin with $\succ$, we must then define indifference by $a \sim_1 b \Leftrightarrow B(a) = B(b)$ and $W(a) = W(b)$ rather than by $a \sim_2 b \Leftrightarrow$ not $a \succ b$ and not $b \succ a$: some of our assumptions, most prominently convexity, would be highly implausible with the second definition. For example, with weak preference given by $\succeq_2 = \succ \cup \sim_2$, the convexity of $\succeq_2$ rules out the canonical case where $a \succ b \Leftrightarrow a \geq b$: $B_{\succeq_2}(a)$ is then never convex. But for $\succeq_1 = \succ \cup \sim_1$, the same $\succ$ leads $B_{\succeq_1}(a)$ always to coincide with the convex set $\{b \in$

$R_+^L: b \succ a\}$ since there are no (a, b) with $a \sim_1 b$. We may in fact reinterpret all of our assumptions on $\succeq$ as assumptions on $\succ \cup \sim_1$ with $\succ$ as the true primitive. See Mandler (2004b) for more on how translate between $\succeq$ and $\succ$ when $\succeq$ is incomplete.

To avoid the trivial incompleteness discussed in the introduction, consider the following ways in which preferences can be substantially incomplete. For $A \subset R^L$, let 'cl A' denote the closure of A, let $\mu(A)$ denote the (Lebesgue) measure of a (Lebesgue) measurable set A, let ∂A denote the boundary of A, and let 'int A' denote the interior of A. When referring to the interior or boundary of A relative to some space $X \neq R_+^L$ and confusion is possible, we write $\text{int}_X A$.

Definition 1 (*substantial incompleteness*). The preference relation $\succeq$ displays

(1) *global incompleteness* if and only if for each $a \in R_+^L$, $B(a) \neq \{a\}$, $W(a) \neq \{a\}$,

$$\text{cl}\, B(a) \cap \text{cl}\, W(a) \subset \{a\},$$

(2) *local incompleteness* if and only if for each $a \in R_+^L$ there is an open $O \supset \{a\}$ such that

$$\text{cl}\, B(a) \cap \text{cl}\, W(a) \cap O \subset \{a\},$$

(3) *measure incompleteness* if and only if for each $a \in R_+^L$ there exists a measurable $A \subset R_+^L$ with $\mu(A) > 0$ that consists of points not ranked with a, that is,

$$A \cap (B(a) \cup W(a)) = \varnothing,$$

(4) *local measure incompleteness* if and only if for each $a \in R_+^L$ and each open $O \supset \{a\}$ there exists a measurable $A \subset O$ with $\mu(A) > 0$ such that

$$A \cap (B(a) \cup W(a)) = \varnothing,$$

(5) *proportionate incompleteness* if and only if for each $a \in R_+^L$ there exists a maximum radius $r > 0$ and a minimum proportion $k > 0$ such that, for any ball β with center a and radius in (0, r), $(\beta \cap R_+^L) \setminus (B(a) \cup W(a))$ contains a measurable set A with $\dfrac{\mu(A)}{\mu(\beta)} > k$.

The meaning of (1) through (4) is mostly self-explanatory. Notice that under global incompleteness, $\text{cl}\, B(a) \setminus \{a\}$ and $\text{cl}\, W(a) \setminus \{a\}$ must be closed relative to $R_+^L \setminus \{a\}$, disjoint, and nonempty, and hence their union cannot equal $R_+^L \setminus \{a\}$. So an open (thus positive measure) set of unranked points remains. With a supplementary condition (see Proposition 1), the same reasoning applies when local incompleteness holds. Condition (5) requires that in all sufficiently small open balls with center a the proportion of points that are unranked relative to a does not fall below some strictly positive lower bound.

Observe that if $\succeq$ is transitive then local or global incompleteness implies that the set of bundles indifferent to a, $B(a) \cap W(a)$, forms a discrete set (since for each $b \sim a$ there would be an open set containing b but no other points indifferent to a or b). So on a compact subset of R_+^L there can be only finitely many bundles indifferent to any a. Surprisingly, some forms of incompleteness lead the unranked bundles to crowd out the indifferent bundles.

Call $\succeq$ *nonisolated* if and only if for all $a \in R_+^L$ and all $\varepsilon > 0$ there exist $b \in B(a) \setminus \{a\}$ and $c \in W(a) \setminus \{a\}$ such that $\|a - b\| < \varepsilon$ and $\|a - c\| < \varepsilon$.

Proposition 1. (1) ⇒ (2) and (5) ⇒ (4) ⇒ (3). If $\succeq$ is nonisolated, then (2) ⇒ (4).

All proofs are in the appendix. To conclude that no other implications among (1) - (5) are possible (without further restrictions on $\succeq$), the following examples suffice. To see that (2) and not (1) are compatible, see Figure 1, which shades the points that are not ranked relative to a and where b is indifferent to a. To see that (5) (and hence (3) and (4)) can hold simultaneously with not (2) and hence not (1), let the set of points not ranked relative to a be a generalized Cantor subset of the shaded area in Figure 1 with positive measure on any neighborhood of a, and let every other shaded point including $\partial B(a)$ be in $W(a)$. Then $\mathrm{cl}\, B(a) \cap \mathrm{cl}\, W(a) = \partial B(a)$, which as pictured violates (2). To see that (3) and not (4) are compatible, see Figure 2. To see that (4), (1) (and hence (2) and (3)) can hold simultaneously with not (5), see Figure 3, where both $\partial B(a)$ and $\partial W(a)$ are differentiable surfaces.

The violation of proportionate incompleteness in Figure 3 also illustrates a more subtle

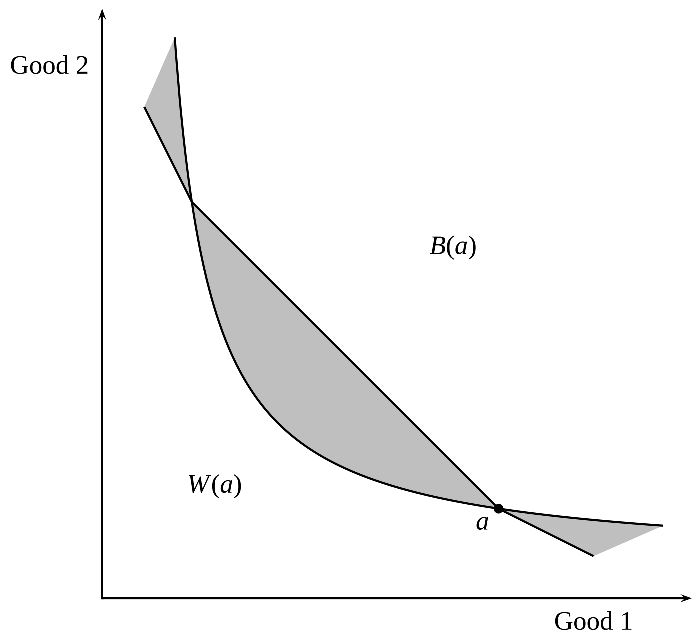

Figure 1: Local but not global incompleteness

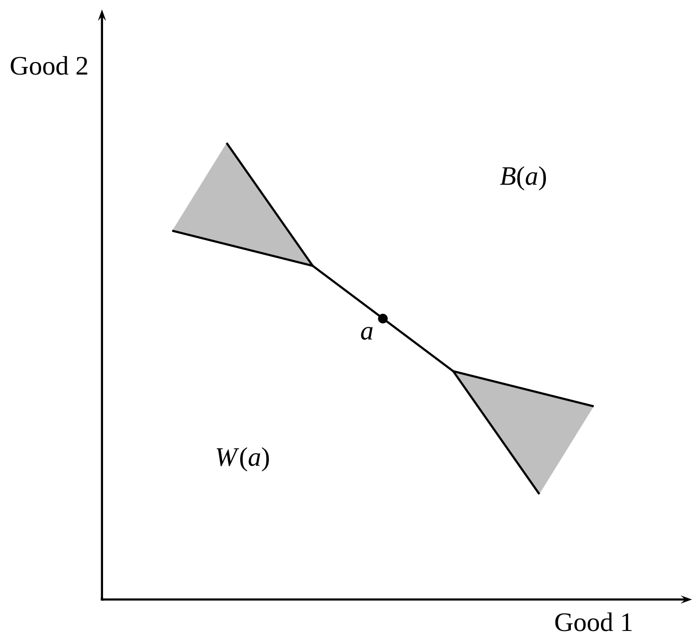

Figure 2: Measure but not local measure incompleteness

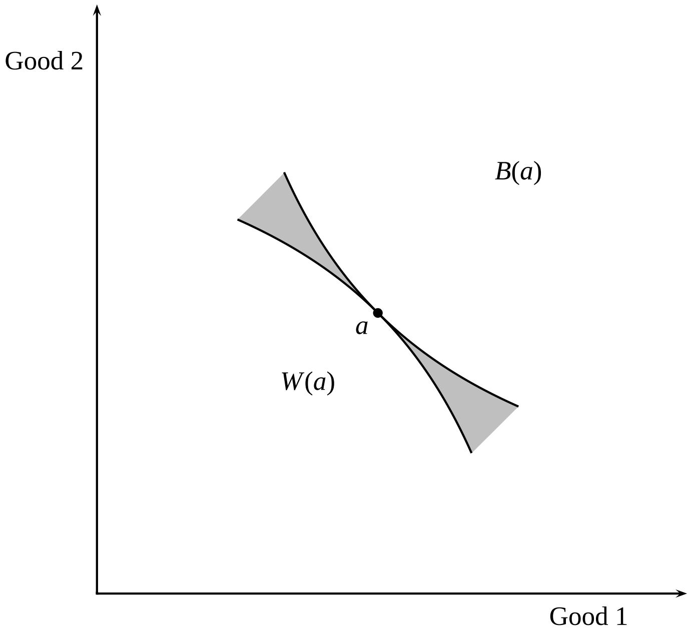

Figure 3: Measure but not proportionate incompleteness

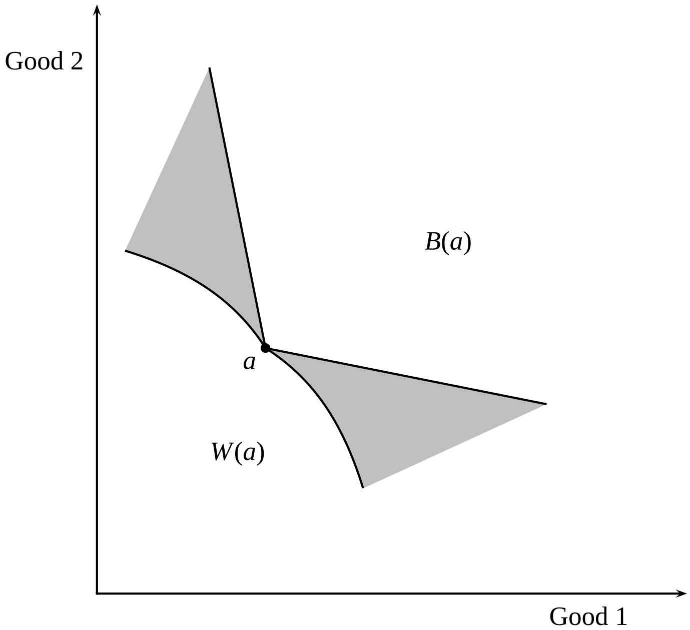

Figure 4: Status quo bias

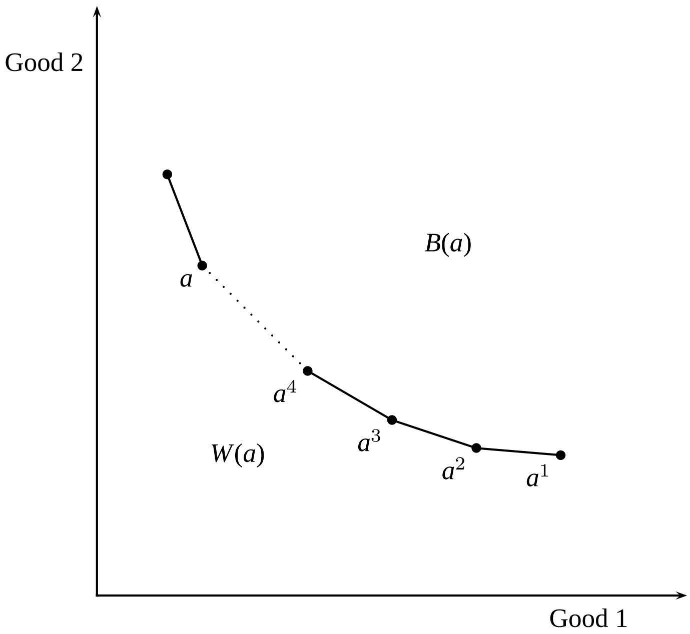

Figure 5: Not smooth and not locally incomplete

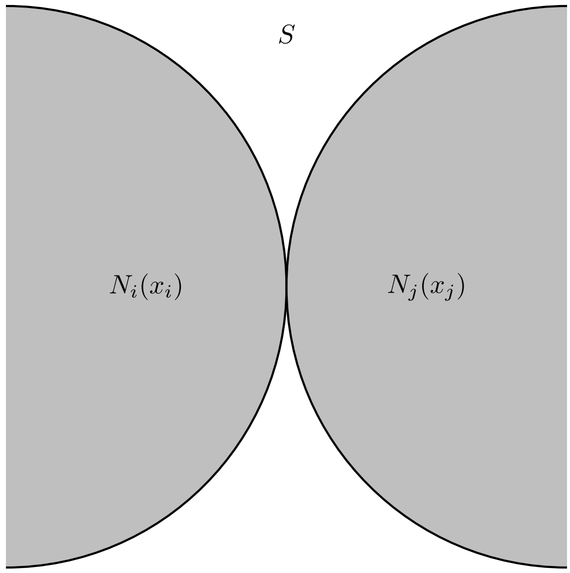

Figure 6: Nontransversal intersection of normal cones

example (compared to the case considered in the introduction) of how preference incompleteness need not affect the dimension of the set of Pareto optima. In Figure 3, as a sequence of open balls with center a shrinks to a itself, the proportion of points unranked to a by $\succeq$ goes to zero; in this limiting sense preferences are locally complete. Since the dimension of the Pareto set turns only on the local properties of preferences, a society of agents with Figure 3 preferences would not experience the dimensional expansion of the Pareto set that incompleteness can lead to.

Status quo bias, however, ensures that proportionate incompleteness obtains. Indeed status quo bias entails all of the above forms of incompleteness except global incompleteness. To formalize status quo bias, let S denote the $L - 1$ dimensional unit circle, $\{p \in R^L\colon \sum_{k \in \mathcal{L}} p_k^2 = 1\}$, and define the *normal cone* of $\succeq$ at $a \in R_+^L$ as $N(a) \equiv \{p \in S\colon p \cdot (b - a) \geq 0$ for all $b \in B(a)\}$. Each $N(a)$ is closed and the unnormalized cone $\{p \in R_+^L\colon p \cdot (b - a) \geq 0$ for all $b \in B(a)\}$ is convex as well as closed. (Terminology warning: $N(a)$ is the negative of what is sometimes called the normal cone.)

Definition 2. The preferences $\succeq$ display *status quo bias* if and only if, for all $a \in R_+^L$, $\text{int}_S\, N(a)$ has dimension $L - 1$.

Status quo bias asserts that every point a has a full-dimensional set of hyperplanes that support $B(a)$. Put differently, on any two-dimensional coordinate plane through a, the set $B(a)$ displays a kink at a (see Figure 4), which is how status quo bias is often modeled in applications. For example, if agents display a WTA-WTP disparity, the minimum amount of money they will accept in exchange for the sacrifice of one unit of some good will be a substantial multiple of the amount of money they will pay for an additional unit of a good. So in the two-dimensional space of money and the good in question, better-than sets $B(a)$ display a kink at any given reference point or endowment a. The equivalent statement put in terms of the normal cone is that $B(a)$ is supported by a set of prices of maximum dimension (i.e., the set of prices has dimension 2, but after normalizing prices to have length one the dimension reduces to 1). Status quo bias as we defined it requires that the same kink or equivalently the same multiplicity of supporting prices is present along any two-dimensional slice of the commodity space.

Outside of the mathematical convenience of stating that $B(a)$ is kinked via the normal cone, our treatment of status quo bias is distinctive in that we meld together the $B(a)$ that an agent reveals at different points a. As in other accounts of status quo bias, we interpret $B(a)$ as the set of bundles an agent will willingly switch to when a is the agent's endowment. Most accounts of status quo bias then go on to label the bundles not in $B(a)$ as dispreferred to a, which then necessitates a separate preferences relation for each endowment point. To preserve the identification of an agent with a single preference relation, we do force this interpretation on the bundles not in $B(a)$. Instead we suppose that a bundle b is identified as dispreferred to a only when the agent in endowed with b and gives it up in favor of a. This view of how $B(a)$ and $W(a)$ are identified is only a matter of interpretation of course; mathematically we simply a posit a single $\succeq$ with $B(a)$ sets that have normal cones of maximum dimension.

Status quo bias does not imply global incompleteness, as Figure 1 illustrates, but we will now see that along with standard economic hypotheses and a technical assumption it implies the other four types of incompleteness in Definition 1. Notice that at one a, a multiplicity of supporting hyperplanes just indicates a kink in $B(a)$ and carries no implication of incompleteness. It is the multiplicity of hyperplanes at every a that carries the implication of incompleteness. To see this, let $\succeq$ be convex, transitive, locally nonsatiated, and satisfy status quo bias and the continuity conditions that $B(a)$ and $W(a)$ are closed for every a. If $\succeq$ were complete, then the boundary of $B(a)$ (which must coincide with the boundary of $W(a)$) would be a convex indifference surface. The intersection of this surface and a (nontangent) hyperplane would define a 'curve' of indifferent points that, because of convexity, is differentiable almost everywhere; hence $B(a)$ could not be kinked at every a, which is inconsistent with status quo bias. Thus with standard background hypotheses status quo bias preferences cannot be complete. But the incompleteness implied by (2) through (5) of Definition 1 is much more extensive that the mere presence of some unranked bundles. To establish that status quo bias implies these broader forms of incompleteness we need an additional technical condition – that the set $N(a)$ changes continuously as a function of a. To see why, consider for example the link between status quo bias and local incompleteness. Suppose status quo bias and transitivity hold and for concreteness that $L = 2$. If the set $N(a)$ were permitted to vary discontinuously as a function of a, there could be a sequence of points $a^n \to a$, with each $a^n \sim a$, such that $N(a^n)$, although always one-dimensional, converges to a single direction. If each a^n lies on the same side of a, then $N(a)$ itself can nevertheless consist of a one-dimensional set (see Figure 5). Thus there could be indifferent bundles arbitrarily near a, which as we have seen is incompatible with local incompleteness if $\succeq$ is transitive. To exclude such pathological cases, we use assumptions from the following family of continuity conditions.

Definition 3. The preferences $\succeq$ have *continuous normals* (resp. *smooth normals*) if and only if $M \equiv \{(a, n) \in R_+^L \times S: n \in N(a)\}$ is a C^0 (resp. C^1) manifold with boundary. The preferences $\succeq$ have *continuous* (resp. *smooth*) *normals at* x_i if and only if, for some open $O \subset R_+^L$ containing x_i, $M \cap (O \times S)$ is a C^0 (resp. C^1) manifold with boundary.

Proposition 2. If $\succeq$ is convex, transitive, locally nonsatiated, has continuous normals, and satisfies status quo bias, then proportionate and local incompleteness obtain.

Since transitivity is included among the assumptions of Proposition 2, the set of bundles indifferent to any a is a discrete set under the same conditions.

Proposition 2 shows only that the proportion of points that are unranked does not go to zero in a neighborhood of a. But not surprisingly, this is the pertinent fact for the dimension of the Pareto optimal allocations.

3. Pareto optimal allocations

We now suppose there is a finite set of agents $\mathbb{I} = \{1, ..., I\}$, each $i \in \mathbb{I}$ described by a preference relation $\succeq_i$. When a notation from the previous section carries an i subscript, it refers to the same set but now defined with $\succeq_i$ rather than $\succeq$.

The economy has an endowment of the L goods, $e \in R^L_{++}$. An *allocation* $x = (x_1, ..., x_n)$ is a point in the $L(I - 1)$-dimensional set of feasible allocations $F = \{x \in R^{LI}_+ : \sum_{i \in \mathbb{I}} x_i = e\}$. Henceforth the 'boundary' or 'interior' of a set of allocations or an 'open set' of allocations are defined relative to F.

Allocation y *Pareto dominates* x if $y_i \succeq_i x_i$ for all $i \in \mathbb{I}$ and $y_i \succ_i x_i$ for some $i \in \mathbb{I}$, and allocation x is *Pareto optimal* if there does not exist an allocation y that Pareto dominates x.

The existence of Pareto optima depends on the continuity of the preferences relations $\succeq_i$, just as with complete preferences. We define $\succeq_i$ to be *upper continuous* if $SW_i(a) = \{b \in R^L_+ : a \succ_i b\}$ is open for all a and *lower continuous* if $SB_i(a) = \{b \in R^L_+ : b \succ_i a\}$ is open for all a. A binary relation $\succ$ on X is *acyclic* if there does not exist a finite set $\{x^1, x^2, ..., x^n\}$, each $x^i \in X$, such that $x^1 \succ x^2 \succ ... \succ x^n \succ x^1$.

Proposition 3. If each $\succeq_i$ is upper continuous and $\succ_i$ is acyclic, then, for any allocation w, $\{z \in F: z$ is Pareto optimal and $w_i \not\succ_i z_i$ for all $i\}$ is nonempty and compact. If in addition for each a and i, $B_i(a)$ is closed, then, for any w, $\{z \in F: z$ is Pareto optimal and $z_i \succeq_i w_i$ for all $i\}$ is nonempty and compact.

The second sentence in Proposition 3 is less appealing than the first since in the absence of completeness it is difficult to find a compelling reason for $B_i(a)$ to be closed. The openness of $SW_i(a)$ relies on any discrete change in an agent's welfare being divisible into smaller nonzero changes, the same rationale one invokes when preferences are complete. But to assume that $B_i(a)$ (or $W_i(a)$) is closed requires that limits of bundles ranked to a be ranked; since incompleteness is permitted, it is not clear how to justify such an assumption.

We now state the welfare theorems in our terminology and highlight that preference completeness is inessential. An allocation x is a *quasiequilibrium* if and only if $\bigcap_{i\in\mathbb{I}} N_i(x_i) \neq \varnothing$ and an *equilibrium* if and only if there is a $p \in S$ such that, for each i, $x_i' \succ_i x_i$ implies $p_i \cdot x_i' > p_i \cdot x_i$. (For a quasiequilibrium, $x_i' \succ_i x_i$ but $p_i \cdot x_i' = p_i \cdot x_i$ for $p \in \bigcap_{i\in\mathbb{I}} N_i(x_i)$ is allowed.) An allocation x is *interior* if and only if $x_i \gg 0$ for all i. Quasiequilibria rather than equilibria provide the more convenient characterization of optimality.

Proposition 4. If each $\succeq_i$ is lower continuous, any interior quasiequilibrium is Pareto optimal. If each $\succeq_i$ is locally nonsatiated and convex, any Pareto optimum is a quasiequilibrium.[1]

The assumptions of Proposition 4 differ from what is needed in an equilibrium characterization of Pareto optimality: to show that an equilibrium is Pareto optimal, local nonsatiation and transitivity are the natural sufficient conditions.

Thus with convex, lower continuous, and locally nonsatiated preferences, we may characterize interior allocations as Pareto optimal if and only if they are quasiequilibria. These conditions on preferences are traditional (convexity) or technical (lower continuity) or mild (local nonsatiation) and in any event allow preferences to be incomplete and/or intransitive. If x is an equilibrium rather than a quasiequilibrium, then local nonindifference would by itself

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[1] To prove the first sentence, let x be a quasiequilibrium, implying there is a $p \in S$ with $p \cdot (b_i - x_i) \geq 0$ for all i and $b_i \in B_i(x_i)$. Now suppose there is a $y \in F$ with $y_i \succeq_i x_i$ for all i and $y_j \succ_j x_j$ for some j. Then $p \cdot (y_i - x_i) \geq 0$ all i. If $p \cdot (y_j - x_j) = 0$ then interiority, lower continuity, and $p \neq 0$ imply there is a y_j' near y_j such that $p \cdot (y_j' - x_j) < 0$, a contradiction. So $\sum_{i\in\mathbb{I}} p \cdot (y_i - x_i) > 0$, which is inconsistent with $\sum_{i\in\mathbb{I}} x_i = \sum_{i\in\mathbb{I}} y_i = e$. We omit the proof of the second sentence; the textbook proof requires only a couple adjustments to avoid transitivity. Notice in this regard that our definition of convexity requires that each $SB_i(a)$ is convex.

(without lower continuity or interiority) imply that x is Pareto optimal. As for the second welfare theorem, convexity and a strengthened version of continuity would by themselves (without local nonsatiation) imply that any interior Pareto optimum is an equilibrium (see Mas-Colell (1974), Shafer and Sonnenschein (1975)). So for the equilibrium version of the welfare theorems completeness and transitivity are again unnecessary.

To argue that most Pareto optima sit amid open sets of Pareto optima, we first identify certain optimal allocations as regular and then show that regularity typically obtains.

Definition 4. A Pareto optimum x is *regular* if and only if $\bigcap_{i \in \mathbb{I}} \mathrm{int}_S N_i(x_i) \neq \varnothing$ and each $\succeq_i$ has continuous normals at x_i.

Given Proposition 4, the following result is immediate.

Proposition 5. If each $\succeq_i$ is lower continuous, then an interior regular Pareto optimum is contained in an open and hence $L(I-1)$-dimensional set of Pareto optima.

In contrast, an economy of I agents with complete, strictly convex, and monotone preferences has a set of Pareto optima of dimension $I-1$ (Arrow and Hahn (1971)).

We can visualize the regular and nonregular optima and see which case is more likely using the concept of transversal intersection. Suppose that $\succeq_i$ displays status quo bias and has smooth normals. In conjunction with status quo bias, the smooth normals assumption implies that the boundary of $N_i(x_i)$ is a smooth $(L-2)$-dimensional surface. Given some ambient manifold $X \subset R^\ell$ (e.g., $X = S$), two sets A and B in X intersect transversally, which we write $A \pitchfork B$, if the affine subspaces that best approximate A and B at any point of common intersection y together span the affine subspace that best approximates X at y.

Consider the simplest economy consisting of two agents i and j. If $N_i(x_i) \pitchfork N_j(x_j)$ and x is optimal – that is, $N_i(x_i) \cap N_j(x_j) \neq \varnothing$ – then x is also a regular optimum. To see this, observe an optimal x will fail to be regular if and only if $N_i(x_i) \cap N_j(x_j) = \partial N_i(x_i) \cap \partial N_j(x_j)$, in which case $\partial N_i(x_i)$ and $\partial N_j(x_j)$ are tangent and hence $N_i(x_i) \pitchfork N_j(x_j)$ does not obtain (see Figure 6). (Notice that a failure of transversality can occur only due to a $n \in \partial N_i(x_i) \cap \partial N_j(x_j)$. If $n \in N_i(x_i) \cap N_j(x_j)$ and $n \in \mathrm{int}_S N_k(x_k)$ for either $k = i$ or $k = j$ then the affine subspace that best approximates $N_k(x_k)$ at n by itself locally spans all of S at n.) Now if we perturb i and j's normal cones then at any given allocation x a failure of transversal intersection will be an exceptional event. But failures of transversal intersection at *some* x can be unavoidable. As x varies along some path in F, $N_i(x_i)$ and $N_j(x_j)$ may switch from being disjoint to intersecting transversally, with nontransversal intersection necessarily occurring at some transition point. Since the qualitative fact that a path changes from $N_i(x_i) \cap N_j(x_j) = \varnothing$ to $N_i(x_i) \cap N_j(x_j) \neq \varnothing$ cannot be perturbed away, nontransversal intersection at some allocation will be unavoidable. The robust way for normal cones to intersect is for the underlying manifolds M_i and M_j (cf. Definition 3) to intersect transversally. To make this precise, place each M_k in the same space $R_+^{LI} \times S$ by setting $M_k = \{(x, n)\colon n \in N_k(x_k)\}$. We will show in the proof of Proposition 6 that the transversality condition $M_i \pitchfork M_j$ typically holds. Then, although a nonregular optimum x is certainly possible, there will always be a nearby optimum x' such that $N_i(x_i') \pitchfork N_j(x_j')$ obtains. (If $N_i(x_i) \pitchfork N_j(x_j)$ failed to hold for an open set of allocations containing x, then the best affine approximations of M_i and M_j at (x, n) would in their n components span only $L - 2$ of the $L - 1$ dimensions in S, thus contradicting $M_i \pitchfork M_j$.) It follows that nonregular optima appear only on the boundary of the set of regular optima. In fact, we will be able to show in addition that the nonregular optima have measure 0.

Analysis of regular optima in economies with three or more agents proceeds along the same lines. If x is such that, for any pair (i, j), $N_i(x_i)$ and $N_j(x_j)$ intersect transversally, for any triple (i, j, k), $N_i(x_i)$ and $(N_j(x_j) \cap N_k(x_k))$ intersect transversally, and so on, then $\bigcap_{i \in \mathcal{I}} N_i(x_i)$

$\neq \varnothing \Rightarrow \bigcap_{i \in \mathcal{I}} \mathrm{int}_S N_i(x_i) \neq \varnothing$: if x is optimal then it is regular.[2] The proof of Proposition 6 then shows by induction that generically every M_i intersects every distinct M_j transversally, every M_i intersects every $M_j \cap M_k$ transversally, and so on. These latter transversality conditions on the M_i will imply that for any nonregular optimum there is a nearby allocation x such the stated transversality conditions on the $N_i(x_i)$ obtain.

We show that the nonregular optima are 0-measure boundary phenomena in Proposition 6 below. The proof proceeds by showing that the above transversality conditions on the M_i hold generically but then in some steps argues directly for results about the nonregular optima (rather than via the transversal intersection of the N_i). Still the above account indicates the underlying geometry.

To formalize the meaning of certain models being typical or generic, we next specify a parameter space of economies.

Definition 5. A *smooth status-quo-bias economy* is an endowment $e \in R^L_{++}$ and a preference profile $(\succeq_1, ..., \succeq_I)$ such that each $\succeq_i$ is locally nonsatiated, lower continuous, convex, has smooth normals, and satisfies status quo bias.

A sequence of preference relations $\succeq^n_i$ that meet the conditions in Definition 5 *converges* to $\succeq_i$ if there is a sequence of C^1 maps $f^n\colon M_i \to R^L_+ \times S$ such that $f^n(M_i) = M^n_i$ (the manifold for $\succeq^n_i$) and f^n converges C^1 uniformly on compacta to the inclusion map of M_i. Using this definition of convergence, we may speak of open and of dense sets of the preference relations in Definition 5 and, using the product topology, of smooth status-quo-bias economies. The convergence of $\succeq^n_i$ requires only that the normals cones at every bundle converge, not that the preference relations (say in the sense of Hausdorff distance) themselves converge. We

[2] Although the intersection of transversal normal cones, e.g., $N_j(x_j) \cap N_k(x_k)$, might not be a manifold, because each $N_j(x_j)$ is a manifold with boundary, the intersection of the boundaries of transversal normal cones will be a manifold, and this suffices for our purposes.

could require this sense of convergence as well, but it is not necessary for our purposes.

Proposition 6. For an open and dense set of smooth status-quo-bias economies, the regular Pareto optima form a nonempty open set PO_R and thus have positive measure. The remaining Pareto optima are contained in the boundary of PO_R and have measure zero.

Since Proposition 6 states results that hold only generically, it might in principle be the case that other desirable properties preferences not entailed by Definition 5 will not hold at all or some of the economies in the identified open and dense set. Reflexivity and weak transitivity in particular are attractive: reflexivity is highly intuitive and weak transitivity is a fundamental rationality property that protects agents from manipulation. To address this point, we show in the proof how to ensure that these properties are satisfied for every economy in the generic set of economies we construct.

It is noteworthy that the tools of differential topology prove so useful in modeling economies of incomplete-preference agents. Although the $B_i(a)$ sets are inherently kinked, smooth techniques can nevertheless be applied to the normal cones that support the $B_i(a)$. See Mas-Colell (1985) for precedents in the theory of production.

4. Partial status quo bias

So far we have considered agents that display status quo bias for all L goods. We now show what it means for status quo bias to hold for a subset of goods and calculate the dimension of the Pareto optimal set that results. Let p_{-k} denote $(p_1, ..., p_{k-1}, p_{k+1},..., p_L)$, and, for $p \in S$ with $p_k \neq 1$, and let $S_k(p)$ denote $\{q \in S\colon \frac{1}{\|q_{-k}\|} q_{-k} = \frac{1}{\|p_{-k}\|} p_{-k}\}$.

Definition 6. Agent *i displays status quo bias for good k at x* if and only if $N_i(x_i)$ is a manifold and whenever p is in the interior of the manifold $N_i(x_i)$ then $\mathrm{int}_{S_k(p)}(N_i(x_i) \cap S_k(p)) \neq \varnothing$. A Pareto optimum x is *regular* if and only if there exists a $p \in S$ such that, for any agent i and good k, if i displays status quo bias for k at x_i then $p \in \mathrm{int}_{S_k(p)}(N_i(x_i) \cap S_k(p))$.

Given an allocation z, let $SQ_i(z)$ equal the number of goods for which i displays status quo bias at z, and let i's *conditional preferences* $\succeq_i(z)$ on the $L - SQ_i(z)$ dimensional subspace of R_+^L consisting of the goods for which i does not display status quo bias be given by $x_i \succeq_i(z)$ y_i if and only if (1) $x_i \succeq_i y_i$ and (2) for any good k, if i displays status quo bias for k then $x_i(k) = y_i(k) = z_i(k)$.

Definition 7. An allocation x has *well-behaved conditional preferences* if and only if, for each i, $\succeq_i(x)$ is strictly convex, monotone, upper and lower continuous, complete and transitive.

Definition 8. An allocation x satisfies *no isolated communities* if and only if for every binary partition $\{\mathbb{I}_1, \mathbb{I}_2\}$ of the set of agents $\mathbb{I}$ there exists a good k such that some $i \in \mathbb{I}_1$ and some $j \in \mathbb{I}_2$ each does not display status quo bias for k at x.

'No isolated communities' (adapted from Smale (1974)) ensures that utility can be continuously transferred among agents using only goods for which agents do not display status quo bias. No isolated communities also implies that every agent has at least two goods for which the agent does not display status quo bias (since status quo bias cannot hold for all but one good). If at some allocation x we fix the consumption levels of goods for which agents display status quo bias, then 'no isolated communities' in conjunction with well-behaved conditional preferences implies that the set of allocations near x that are Pareto optimal subject to these constraints has dimension $I - 1$ (see Arrow and Hahn (1971), or Mas-Colell (1985) for a detailed treatment).

Proposition 7. If each i's preferences are convex and lower continuous, and if the interior regular optimum x has well-behaved conditional preferences and satisfies 'no isolated communities,' then x is contained in a set of optima of dimension $I - 1 + \sum_{i \in \mathbb{I}} SQ_i(x)$.

5. Utilitarianism with incomplete preferences

Preference incompleteness or status quo bias can lead so many allocations to be Pareto optimal that the Pareto criterion loses much of its usefulness. But the Pareto criterion is only one way to make welfare judgments. We therefore ask: does preference incompleteness always make welfare decisions problematic or is the Pareto criterion particularly likely to label large sets of allocations as optimal? We argue for the latter conclusion by examining briefly the capacity of utilitarianism – the second-most popular economic welfare criterion – to discriminate among allocations. When classical utilitarianism faces agents with complete and strictly convex preferences, it designates one allocation as a global optimum. When preferences are incomplete, a utilitarian planner cannot always give advice that is this sweeping, but even the worst case compares favorably with Pareto optimality. The issues are mathematically simple and so we can be concise.

Call a set of functions Z, where each $z \in Z$ maps some common set Y to R, a *cardinal set* if: $z \in Z \Leftrightarrow$ for all $z' \in Z$, there exist $a > 0$ and b such that $z = az' + b$. Given a Cartesian product of n cardinal sets $U = U_1 \times \ldots \times U_n$, call $\hat{U} \subset U$ a *cardinal selection from U* if: $(u_1, \ldots, u_n) \in \hat{U} \Leftrightarrow$ for all $(u_1' \ldots, u_n') \in \hat{U}$, there exist $a > 0$ and $(b_1, \ldots, b_n)$ such that $u_i = au_i' + b_i$ for $i = 1, \ldots, n$.

With this terminology, we can describe classical utilitarianism as beginning with a cardinal set of utilities U_i for each agent i, where any $u_i \in U_i$ is a utility function on the consumption set R_+^L. A utilitarian planner specifies a cardinal selection from $U_1 \times \ldots \times U_I$ that we label W. The requirement that all profiles in W are rescaled by the same 'units' term a means that the utilitarian planner knows how to translate between the utility units of any pair of agents. Cardinality thus arises at two levels: in individual utility sets and in the planner's selection of comparable utilities.[3] The planner judges an allocation x to be weakly superior to y

[3] Modeling utilitarianism via cardinal sets of utilities or utility transformations is traditional; see Sen (1970), d'Aspremont and Gevers (1977), Roberts (1980), Bossert and

if, for any (and therefore all) $(u_1, ..., u_I) \in W$, $\sum_{i \in \mathcal{I}} u_i(x_i) \geq \sum_{i \in \mathcal{I}} u_i(y_i)$. An optimum x in a feasible set F satisfies $\sum_{i \in \mathcal{I}} u_i(x_i) \geq \sum_{i \in \mathcal{I}} u_i(y_i)$ for all $y \in F$.

To apply this method to potentially incomplete preferences, suppose that each agent is able to make a full cardinal set of utility judgments for each good taken separately. As with complete preferences, the planner takes these sets as data, but now there are L sets per agent, one set for each good. Incompleteness occurs when agents do not know how to compare the utilities of goods and hence cannot always rank bundles that trade-off the consumption of different goods. When agents are unable to judge these trade-offs, it would be inappropriate to endow the planner with that ability. But since agents possess cardinal judgments about the strength of preference for any single good k, a utilitarian planner presumably *would* be able to compare the utilities of different agents for k, that is, make a cardinal selection from agents' sets of utilities for k. Indeed it is presumably easier to interpersonally compare utilities for a single good than for bundles. And good-by-good interpersonal comparisons are enough for utilitarianism to be reasonably discriminating.

Formally, agent i is endowed with L cardinal sets of functions $V_i[\![1]\!], ..., V_i[\![L]\!]$. Each function $v_i[\![k]\!]$: $R_+^L \to R$ in $V_i[\![k]\!]$ indicates the utility of good k but is a function of the consumption level of the other $L - 1$ goods as well since they might be complements or substitutes for k. A prominent special case, which we call *separability*, occurs if each $v_i[\![k]\!] \in V_i[\![k]\!]$ varies only as a function of $x_i[\![k]\!]$: that is, for any $k \in \mathcal{L}$ and $x_i[\![k]\!] \in R_+$, $v_i[\![k]\!] \in V_i[\![k]\!]$ is constant on $\{y_i \in R_+^L: y_i[\![k]\!] = x_i[\![k]\!]\}$. Separability is somewhat more plausible when preferences are incomplete than additive separability is in ordinary utility theory. Each i aggregates these sets of utilities by a $V_i \subset V_i[\![1]\!] \times ... \times V_i[\![L]\!]$, whose typical element is a L-tuple of functions $v_i = (v_i[\![1]\!], ..., v_i[\![L]\!])$. Given any $v_i \in V_i$, we define a utility function on R_+^L by summing commodity coordinates: $\sum_{k \in \mathcal{L}} v_i[\![k]\!]$. We suppose that agent i sees each such

Weymark (1996).

$\Sigma_{k \in \mathscr{L}} v_i [\![k]\!]$ as an equally legitimate way to evaluate bundles and hence prefers x_i to y_i only when all such evaluations rank x_i higher than y_i. So henceforth i's preferences $\succeq_i$ on R_+^L will be given by: $x_i \succeq_i y_i$ if and only if $\Sigma_{k \in \mathscr{L}} v_i [\![k]\!](x_i) \geq \Sigma_{k \in \mathscr{L}} v_i [\![k]\!](y_i)$ for all $v_i \in V_i$. Note that $\succeq_i$ must be transitive. If V_i is a cardinal selection from $V_i [\![1]\!] \times ... \times V_i [\![L]\!]$, then $\{ \Sigma_{k \in \mathscr{L}} v_i [\![k]\!]$: $v_i \in V_i \}$ forms a cardinal set of functions and so we would return to the complete ordinal preferences and full cardinality data used by a utilitarian planner. But when V_i consists of a larger set of functions (in the extreme, all of $V_i [\![1]\!] \times ... \times V_i [\![L]\!]$), i would not be able to cardinally compare his/her strength of preference across goods and $\succeq_i$ would be incomplete. Although we will not use this assumption, it is natural to suppose that any V_i equals a union of cardinal selections from $V_i [\![1]\!] \times ... \times V_i [\![L]\!]$ – any smaller V_i would throw away v_i without changing $\succeq_i$. Although it is in principle restrictive that $\succeq_i$ is derived by summing the $v_i [\![k]\!]$, considerable flexibility remains. We may admit any $\succeq_i$ that has a utility representation u_i: set $V_i [\![k]\!] = \{ a u_i + b$: $a \in R_{++}$, $b \in R \}$ for each good k and let V_i be any cardinal selection from $V_i [\![1]\!] \times ... \times V_i [\![L]\!]$. At the other end of spectrum, we can admit the extreme incomplete preference relation $\succeq_i$ that ranks $y_i \succeq_i z_i$ if and only if $y_i \geq z_i$: set $V_i [\![k]\!] = \{ a g_i [\![k]\!] + b$: $a \in R_{++}$, $b \in R \}$, where $g_i [\![k]\!]$: $R_+ \to R$ is increasing, and $V_i = V_i [\![1]\!] \times ... \times V_i [\![L]\!]$.

The convex status-quo-bias preferences considered in sections 2-4 readily appear under this set-up: let each $V_i [\![k]\!]$ contain only concave functions, fix a nontrivial interval of weights $A_i [\![k]\!] \subset R_{++}$ for each k and a reference $(\bar{v}_i [\![1]\!], ..., \bar{v}_i [\![L]\!]) \in V_i [\![1]\!] \times ... \times V_i [\![L]\!]$, and let $(v_i [\![1]\!], ..., v_i [\![L]\!]) \in V_i$ if and only if, for all k, $v_i [\![k]\!] = \alpha \bar{v}_i [\![k]\!] + \beta$ for some $\alpha \in A_i [\![k]\!]$ and $\beta \in R$.

A utilitarian planner takes as data for each agent i the sets $V_i [\![1]\!], ..., V_i [\![L]\!]$, and V_i, and then aggregates agent utilities for each good k, just as a classical planner would aggregate agent utilities for bundles of L goods. With access to a full set of cardinal utilities for each agent for each good k, a utilitarian planner should able to specify for each k a cardinal selection from $V_1 [\![k]\!] \times ... \times V_I [\![k]\!]$. We denote this selection by $W[\![k]\!]$, which has as its typical element an I-tuple of agent utilities for good k, $v[\![k]\!] \equiv (v_1 [\![k]\!], ..., v_I [\![k]\!])$. The planner then builds one utilitarian ranking for each good k: the kth good ranking judges allocation x to be weakly superior to y if, for any (and hence all) $v[\![k]\!] \in W[\![k]\!]$, $\sum_{i \in \mathbb{I}} v_i[\![k]\!](x_i) \geq \sum_{i \in \mathbb{I}} v_i[\![k]\!](y_i)$.

The basic cardinal utility building blocks that we have posited are comparable to what a classical utilitarian begins with, except that in the present setting cardinal utilities are defined separately for each good. The planner can therefore make independent good-by-good decisions about how to compare the welfare of different individuals (though, not surprisingly, we will see that this latitude disappears when preferences are complete). Once these utility comparisons have been made, traditional utilitarian results follow: $\sum_{i \in \mathbb{I}} v_i[\![k]\!]$ will be maximized by transferring goods from low-marginal-utility agents to high-marginal-utility agents. We stick to the utilitarian practice of adding utilities to underscore this common ground: the redistributionist logic of utilitarianism applies unaltered to incomplete preferences. Note though that Proposition 8 below would continue to hold if instead we concavely aggregated the I agent utility functions for k.

The utilitarian ranking for good k compares the utility that the I agents ascribe to good k. Due to the possibility of nonseparability, the $v_i[\![k]\!] \in V_i[\![k]\!]$ may be functions of the remaining goods and hence the allocation of these goods can affect the planner's ranking for k. Conversely the allocation of k can affect the rankings for the other goods. So for example an allocation x that differs from y only in the good k coordinates might be judged superior to y according the kth good ranking but inferior according to one of the other good rankings.[4] In the separable case this particular ambiguity cannot arise. But even with separability there will obviously be pairs of allocations differing in multiple coordinates that can be ranked differently by separate commodity rankings.

One way to proceed would be to aggregate the $\sum_{i \in \mathbb{I}} v_i[\![k]\!]$ across goods, perhaps via a

[4] That is, for x and y with $x_i[\![\ell]\!] = y_i[\![\ell]\!]$ for $i \in \mathbb{I}$ and $\ell \neq k$, we could have for some $h \neq k$ both $\sum_{i \in \mathbb{I}} v_i[\![k]\!](x_i) > \sum_{i \in \mathbb{I}} v_i[\![k]\!](y_i)$ and $\sum_{i \in \mathbb{I}} v_i[\![h]\!](y_i) > \sum_{i \in \mathbb{I}} v_i[\![h]\!](x_i)$ where $v[\![k]\!] \in W[\![k]\!]$ and $v[\![\ell]\!] \in W[\![\ell]\!]$.

weighted sum $\Sigma_{k \in \mathcal{L}} \delta_k [\Sigma_{i \in \mathcal{I}} v_i [\![k]\!]]$, to arrive at a single welfare function that will order any pair of allocations. But we wish to avoid having planners impose judgments on the trade-offs that the agents themselves are unable to make. Instead we require agreement among the kth good utilitarian rankings before a change in allocations qualifies as an improvement, or, put in terms of optimality, that an allocation x is optimal if any change that is an increase for one of the kth good utilitarian rankings is a decrease for one of the other commodity rankings. Recall that an allocation is by definition in the economy's feasible set $F = \{y \in R_+^{LI}\colon \Sigma_{i \in \mathcal{I}} y_i = e\}$.

Definition 9. Allocation x is a *utilitarian optimum* if and only if, for every allocation y such that $\Sigma_{i \in \mathcal{I}} v_i [\![k]\!](y_i) > \Sigma_{i \in \mathcal{I}} v_i [\![k]\!](x_i)$ for some $k \in \mathcal{L}$ and $v[\![k]\!] \in W[\![k]\!]$, there is a $\ell \in \mathcal{L}$ and $v[\![\ell]\!] \in W[\![\ell]\!]$ such that $\Sigma_{i \in \mathcal{I}} v_i [\![\ell]\!](y_i) < \Sigma_{i \in \mathcal{I}} v_i [\![\ell]\!](x_i)$.

To address the number of utilitarian optima, we assume that each $V_i [\![k]\!]$ contains only functions that are strictly concave on the coordinate subspace on which they are nonconstant. Formally, *strict coordinate concavity* obtains if, for each $i \in \mathcal{I}$ and $k \in \mathcal{L}$, there is a nonempty set of coordinates $K \subset \mathcal{L}$ such that, for all $x_i \in R_+^L$, $v_i [\![k]\!] \in V_i [\![k]\!]$ is strictly concave on $\{y_i \in R_+^L\colon y_i [\![\ell]\!] = x_i [\![\ell]\!] \text{ for } \ell \notin K\}$ and constant on $\{y_i \in R_+^L\colon y_i [\![\ell]\!] = x_i [\![\ell]\!] \text{ for } \ell \in K\}$. If separability and strict coordinate concavity hold then for any k there is a $\bar{x} [\![k]\!] = (\bar{x}_1 [\![k]\!], ..., \bar{x}_I [\![k]\!])$ such that any maximizer of $\Sigma_{i \in \mathcal{I}} v_i [\![k]\!]$, where $v[\![k]\!] \in W[\![k]\!]$, has $\bar{x} [\![k]\!]$ as its good k coordinates. Hence $(\bar{x} [\![1]\!], ..., \bar{x} [\![L]\!])$ is the unique utilitarian optimum. At the other extreme, suppose in addition to strict coordinate concavity that $v_i [\![k]\!] \in V_i [\![k]\!]$ is monotone (i.e., $v_i [\![k]\!]$ represents a monotone preference relation on R_+^L) for each $i \in \mathcal{I}$ and $k \in \mathcal{L}$. Since x is a utilitarian optimum if and only if x is Pareto optimal for an economy with endowment e and the L utilities $\Sigma_{i \in \mathcal{I}} v_i [\![k]\!]$, $k \in \mathcal{L}$, where $v[\![k]\!] \in W[\![k]\!]$, the utilitarian optima are a $L - 1$ dimensional set. Thus we have:

Proposition 8. If strict coordinate concavity obtains then separability implies there is a unique utilitarian optimum, and monotonicity for $v_i [\![k]\!] \in V_i [\![k]\!]$ for $i \in \mathcal{I}$ and $k \in \mathcal{L}$ implies the utilitarian optima form a set of dimension $L - 1$.

As long as strict coordinate concavity holds, $L - 1$ is the worst case, that is, the maximum dimension of the utilitarian optima. If we were to relax monotonicity so that utilities were increasing only on a subset of goods, then 'isolated communities' could arise in which the utilities $\sum_{i \in \mathbb{I}} v_i [\![k]\!]$, $k \in \mathscr{L}$, could be partitioned into families that are monotone on disjoint sets of goods (see Smale (1974) and section 4). The dimension of the utilitarian optima would then drop below $L - 1$. Indeed separability is an example of 'isolated communities': each $\sum_{i \in \mathbb{I}} v_i [\![k]\!]$ varies only as a function of the goods $x_1 [\![k]\!], ..., x_I [\![k]\!]$ that do not affect the remaining $\sum_{i \in \mathbb{I}} v_i [\![\ell]\!]$, $\ell \neq k$. But even in the worst $L - 1$ dimensional case, the set of utilitarian optima has not undergone the $L(I - 1)$ explosion of dimensionality we saw for Pareto optimality (Proposition 5). If, as one presumes when markets are competitive, the number of individuals is larger than the number of goods, $I > L$, then the dimension of utilitarian optima is smaller than the dimension of Pareto optima when preferences are complete.

To conclude we deal with the possibility that a utilitarian optimum need not be Pareto optimal. If we eliminate this problem by fiat – by simply requiring that a planner choose $W[\![k]\!]$, $k \in \mathscr{L}$, and a utilitarian optimum x so that x is Pareto optimal – then we restrict the admissible allocations and hence do not expand any multiplicity of optima. But it will fill out our picture to see how a planner should choose cardinal selections $W[\![k]\!]$ to avoid Pareto suboptimality.

Definition 10. The cardinal selections $(W[\![k]\!])_{k \in \mathscr{L}}$ are *Pareto compatible* if and only if there exist $(v_1 [\![k]\!], ..., v_I [\![k]\!]) \in W[\![k]\!]$, $k \in \mathscr{L}$, such that $(v_i [\![1]\!], ..., v_i [\![L]\!]) \in V_i$, $i \in \mathbb{I}$.

As preferences become more incomplete, Pareto compatibility becomes a less demanding restriction. If $V_i = V_i [\![1]\!] \times ... \times V_i [\![L]\!]$ for each i then *any* $(W[\![k]\!])_{k \in \mathscr{L}}$ is Pareto compatible. At the other extreme, suppose that each V_i is a cardinal selection from $V_i [\![1]\!] \times ... \times V_i [\![L]\!]$ and hence that each $\succeq_i$ is complete. Then, given one $W[\![k]\!]$, and if each $V_i [\![k]\!]$ contains nonconstant functions, only one selection of the remaining $W[\![\ell]\!]$, $\ell \neq k$, will be consistent with Pareto compatibility.[5] As expected, with complete preferences, the planner's latitude to make interpersonal comparisons that vary independently by good disappears. We say that $V_i[\![k]\!]$ has *regular minima* if any $v_i[\![k]\!] \in V_i[\![k]\!]$ has at most one local minimum which if it exists is a global minimum.

Proposition 9. If separability obtains, each $\succeq_i$ is lower continuous, each $V_i[\![k]\!]$ has regular minima, and $(W[\![k]\!])_{k \in \mathscr{L}}$ is Pareto compatible, then any utilitarian optimum is Pareto optimal.

Following the proof in the appendix, we show that if $V_i = V_i[\![1]\!] \times ... \times V_i[\![L]\!]$ and $v_i[\![k]\!] \in V_i[\![k]\!]$ is concave for each i and k, then again any utilitarian optimum is Pareto optimal.

6. Conclusion

If incompleteness of preference takes the form of status quo bias, the set of Pareto optimal allocations can be very large. In the polar case where a willingness-to-accept/willingness-to-display disparity occurs in every direction of movement, the dimension of the Pareto optima will equal the entire dimension of the commodity space. Utilitarianism in contrast knocks the dimension of indeterminacy down to $L - 1$ at worst and at best identifies a unique optimum.

The distinctive feature of utilitarianism under incomplete preferences is that it proceeds good by good: when aggregating utilities, the planner can independently place weights on the

[5] Given arbitrary $(v_1[\![k]\!], ..., v_I[\![k]\!]) \in W[\![k]\!]$, there exists $(v_1[\![\ell]\!], ..., v_I[\![\ell]\!]) \in W[\![\ell]\!]$, $\ell \neq k$, such that such that $(v_i[\![1]\!], ..., v_i[\![L]\!]) \in V_i$, $i \in \mathbb{I}$: take the $(\hat{v}_1[\![\ell]\!], ..., \hat{v}_I[\![\ell]\!]) \in W[\![\ell]\!]$, $\ell \in \mathscr{L}$, given by Pareto compatibility, and set $(v_1[\![\ell]\!], ..., v_I[\![\ell]\!]) \in W[\![\ell]\!]$, $\ell \neq k$ equal to $a(\hat{v}_1[\![\ell]\!], ..., \hat{v}_I[\![\ell]\!]) \in W[\![\ell]\!]$, $\ell \neq k$, where a solves $a(\hat{v}_1[\![k]\!], ..., \hat{v}_I[\![k]\!]) + (b_1, ..., b_I) = (v_1[\![k]\!], ..., v_I[\![k]\!])$. Fixing some $(\bar{v}_i[\![1]\!], ..., \bar{v}_i[\![L]\!]) \in V_i$ for each i, V_i being a cardinal selection implies that for any $(\tilde{v}_i[\![1]\!], ..., \tilde{v}_i[\![L]\!]) \in V_i$ there exist $a_i > 0$ and $(b_{i1}, ..., b_{iL})$ such that $\tilde{v}_i[\![\ell]\!] = a_i \bar{v}_i[\![\ell]\!] + b_{i\ell}$ for $\ell \in \mathscr{L}$. If $V_i[\![k]\!]$ does not contain constant functions, a unique (a_i, b_{ik}) solves $\tilde{v}_i[\![k]\!] = a_i \bar{v}_i[\![k]\!] + b_{ik}$. Thus for $(\tilde{v}_i[\![1]\!], ..., \tilde{v}_i[\![L]\!]) = (v_i[\![1]\!], ..., v_i[\![L]\!])$, we conclude, for any $\ell \in \mathscr{L}$, that $v_i[\![\ell]\!] = \bar{a}_i \bar{v}_i[\![\ell]\!] + b_{i\ell}$ for some $b_{i\ell}$, where $\bar{a}_i$ solves $v_i[\![k]\!] = \bar{a}_i \bar{v}_i[\![k]\!] + b_{ik}$. Thus $(\bar{a}_i \bar{v}_i[\![\ell]\!] + b_{i\ell})_{i=1}^{I} \in W[\![\ell]\!]$ and hence $W[\![\ell]\!] = \{(\alpha \bar{a}_i \bar{v}_i[\![\ell]\!] + b_{i\ell})_{i=1}^{I} : \alpha \in R_+, (b_{i\ell})_{i=1}^{I} \in R^I\}$.

utility generated by the different goods consumed by a single individual. In complete-preference welfare economics in contrast only one weighting is made for every individual. This single weighting can lead to sweeping conclusions; in market settings, policymakers do not need to and should not make good-by-good distribution decisions, they just need to decide whom to redistribute wealth to. But with incomplete preferences, room opens up for policymakers to judge separately for each good *k* how much an agent's consumption of *k* will contribute to the social good.

Appendix.

Proof of Proposition 1. The implications in the first sentence follow immediately from the definitions. Assume nonisolation and (2), and let $D = O \setminus \{a\}$ where O is the open subset given in (2). Since $\mathrm{cl}\, B(a) \cap D$ and $\mathrm{cl}\, W(a) \cap D$ are closed in D, disjoint, and (by nonisolation) nonempty, their union cannot equal D. Hence the complement of $(\mathrm{cl}\, B(a) \cap D) \cup (\mathrm{cl}\, W(a) \cap D)$ in D must be nonempty. Since this set is also open, it has positive measure. ■

Proof of Proposition 2. Local incompleteness. To show that there is an open O containing a such that $\partial B(a) \cap \partial W(a) \cap O \subset \{a\}$, suppose to the contrary that there is a sequence $\{b^t \in R_+^L\}$ such that $b^t \to a$, $b^t \neq a$, and $b^t \in \partial B(a) \cap \partial W(a)$. In the next paragraph, we use the fact (implied by convexity, transitivity, and local nonsatiation) that $b^t \in \partial B(a)$ to find a subsequence of $\{b^t\}$ that lies near to the graph of a convex function *f* restricted to a line in its domain. Since the convexity of *f* implies that its one-sided directional directives converge, the subsequence of $\{b^t\}$ approximates a line near a. Then we take points near b^t in $W(a)$ and see that the directions in the normal cones of these points must be arbitrarily near the orthogonal complement of the line defined by $\{b^t\}$, and this is inconsistent with $\succeq$ simultaneously satisfying status quo bias and having continuous normals.

Status quo bias implies there is a $\bar{p} \in \mathrm{int}_S N(a)$. There must then be a nonempty open ball $\beta \subset R_+^L$ with center a and radius r such that $F(c) = \{\alpha \in R : c + \alpha \bar{p} \in \partial B(a)\} \neq \varnothing$ for all c

$\in H \equiv \beta \cap \{c \in R^L_+ \colon \bar{p} \cdot c = \bar{p} \cdot a\}$. Define $f\colon H \to R$ by $f(c) = \min F(c)$. Again because $\bar{p}$ lies in the interior of $N(a)$, for b^t near a there is a unique $c^t \in H$ such that $c^t + f(c^t)\bar{p} = b^t$, and so we identify any b^t near a with this $c^t \in H$. (Notice that the graph of f is an affine translation of $\partial B(x)$.) For any $\varepsilon > 0$, we may cover $\{c \in H\colon \|c - a\| \le r\}$ by a finite number of cones with apex a and whose base at the boundary of β has diameter $\le \varepsilon$. More precisely, each cone is the convex hull of $\{a\}$ and some $C \subset \{y \in H\colon \|c - a\| = r\}$ such that if $c, c' \in C$ then $c - c' \le \varepsilon$. For any $\varepsilon > 0$ therefore we may select a subsequence of $\{c^t\}$ that accumulates in one of these cones, and letting $\varepsilon \to 0$, a further subsequence $\{d^t\}$ and limit direction $\bar{d} - a \in S$ such that $\frac{1}{\|d^t - a\|}(d^t - a)$ converges to $(\bar{d} - a)$. Since f is convex and $a \in$ int domain f, the one-sided directional derivative of f at a is a continuous function of its direction and therefore the difference quotient $\frac{f(d^t) - f(a)}{\|d^t - a\|}$ converges to the one-sided directional derivative of f at a in direction $\bar{d}$, say $f'(a, \bar{d})$ (see, e.g., Rockafeller (1970, §23)). If we define $\bar{v} = \bar{d} + f'(a, \bar{d})\bar{p}$ and let $\{v^t = d^t + f(d^t)\bar{p}\}$ denote the subsequence of $\{b^t\}$ that corresponds to $\{d^t\}$, we conclude that the direction $\frac{1}{\|v^t - a\|}(v^t - a)$ converges to $\frac{1}{\|\bar{v} - a\|}(\bar{v} - a)$. Thus if $\{q^t \in S\}$ is a sequence of directions with each q^t orthogonal to $v^t - a$ and $\bar{q}$ is an accumulation point of $\{q^t\}$ then $\bar{q}$ lies in the orthogonal complement of $\bar{v} - a$: $\bar{q} \in (\bar{v} - a)^{\text{orth}} \equiv \{q \in S\colon q \cdot (\bar{v} - a) = 0\}$. It is straightforward to show that any $\varepsilon > 0$ there are T and T' where for $t > T$ and $t' > T'$ such that $\frac{1}{\|v^{t'} - a\|}(v^{t'} - a)$ and $\frac{1}{\|v^t - v^{t'}\|}(v^t - v^{t'})$ are both within ε of $\frac{1}{\|\bar{v} - a\|}(\bar{v} - a)$ (first select T so that for $\hat{t} > T$ $\frac{1}{\|v^{\hat{t}} - a\|}(v^{\hat{t}} - a)$ is within ε of $\frac{1}{\|\bar{v} - a\|}(\bar{v} - a)$ and then choose T' so that $v^{t'}$ is near a). Then for any $q \notin (\bar{v} - a)^{\text{orth}}$ there is a T' such that, for $t' > T'$, $q \notin \eta(v^{t'}, a) \equiv \{p \in S\colon p \cdot (a' - v^{t'}) \ge 0 \text{ for all } a' \in B(a)\}$.

Given some $v^{t'} \in \partial B(a) \cap \partial W(a)$, there is a sequence $\{w^n(v^{t'})\}$ with $w^n(v^{t'}) \in W(a)$ and $w^n(v^{t'}) \to v^t$. By the transitivity of $\succeq$, $B(w^n(v^{t'})) \supset B(a)$, and so if $q \notin \eta(v^{t'}, a)$ then $q \notin N(w^n(v^{t'}))$ for n sufficiently large. So for any $q \notin (\bar{v} - a)^{\text{orth}}$ there is a T' and N such that $q \notin N(w^n(v^{t'}))$ for $t' > T'$ and $n > N$. It follows that for any $\varepsilon > 0$ there exists $w^n(v^{t'})$ such that $q \in N(w^n(v^{t'}))$ implies $\min\{\|q - \bar{q}\|\colon \bar{q} \in (\bar{v} - a)^{\text{orth}}\} < \varepsilon$. This violates smoothness however since this minimum converges to 0 as $w^n(v^{t'})$ approaches a and yet $\text{int}_S N(a)$ has dimension $L - 1$.

Proportionate incompleteness. For any $\bar{c} \in H$, with H defined as before, let $L(\bar{c})$ denote the line segment $\{\lambda \bar{c} + (1 - \lambda)a\colon \lambda \in R_+\} \cap H$. Let F and f be defined as before except restricted to $L(\bar{c})$: for $c \in L(\bar{c})$, $F(c) = \{\alpha \in R\colon c + \alpha \bar{p} \in \partial B(a)\}$ and $f\colon L(\bar{c}) \to R$ is given by $f(c) = \min F(c)$. In addition, define $G(c) = \{\alpha \in R\colon c + \alpha \bar{p} \in \partial W(a)\}$ and let $g\colon L(\bar{c}) \to R$ be given by $g(c) = \max G(c)$ when $G(c) \neq \varnothing$ and $g(c) = -\|c - a\|$ when $G(c) = \varnothing$. Observe that for $c \in L(\bar{c})$ the points in $\{c + \alpha \bar{p}\colon \alpha \in R\}$ that are unranked relative to a (i.e., not in $B(a) \cup W(a)$), contain a line of length segment equal to at least $f(c) - g(c)$. Let β_O denote some closed ball with center a and radius $\bar{r} > 0$ that is contained in the O given by local incompleteness; thus $f(c) - g(c) > 0$ for $c \in (L(\bar{c}) \cap \beta_O)\backslash\{a\}$. Let us define

$$h(\bar{c}) \equiv \{\frac{\|f(c) - g(c)\|}{\|c - a\|}\colon c \in (L(\bar{c}) \cap \beta_O)\backslash\{a\}\}.$$

To see that $\liminf h(\bar{c}) > 0$, suppose to the contrary that there is $\{c^t\} \subset (L(\bar{c}) \cap \beta_O)\backslash\{a\}$ with $c^t \to a$ and $\frac{\|f(c^t) - g(c^t)\|}{\|c^t - a\|} \to 0$, and consider the corresponding sequence $v^t = c^t + f(c^t)\bar{p}$. Then, as previously, there is a sequence of pairs (t, t') such that, for any $\varepsilon > 0$, $\frac{1}{\|v^{t'} - a\|}(v^{t'} - a)$ and $\frac{1}{\|v^t - v^{t'}\|}(v^t - v^{t'})$ lie within ε of some fixed limit direction $\frac{1}{\|\bar{v} - a\|}(\bar{v} - a)$ for all (t, t') sufficiently large. Since $\frac{\|f(c^t) - g(c^t)\|}{\|c^t - a\|} \to 0$, $G(c^t) \neq \varnothing$ for all t sufficiently large and so there must be a sequence $\{w^n\} \subset W(a)$ (hence with $B(w^n) \supset B(a)$) such that $\frac{\|v^t - w^n\|}{\|v^t - a\|} \to 0$ for n and t large. Thus if $q \notin (\bar{v} - a)^{\text{orth}}$ then $q \notin N(w^n)$ for n sufficiently large, again leading to a violation of the continuous normals assumption.

Let $P(\bar{c}) = \{b \in R^L_+\colon b = c + \alpha \bar{p}$ for some $c \in L(\bar{c}), \alpha \in R\}$. Since $\liminf h(\bar{c}) > 0$, there is, for each $\bar{c} \in H\backslash\{a\}$, a radius $r_{\bar{c}} > 0$ and a proportion $k > 0$ such that, for any ball $\beta_{\bar{c}}$ with center a and radius in $(0, r_{\bar{c}})$, $(\beta_{\bar{c}} \cap P(\bar{c}))\backslash(B(a) \cup W(a))$ contains a measurable set A with $\frac{\mu_{\bar{c}}(A)}{\mu_{\bar{c}}(\beta_{\bar{c}} \cap P(\bar{c}))} > k$, where $\mu_{\bar{c}}$ denotes Lebesgue measure on $P(\bar{c})$. Since local incompleteness implies that $f(c) - g(c)$ is bounded away from 0 for $c \in (L(\bar{c}) \cap \beta_O)\backslash\beta_{\bar{c}}$, there is a $k' > 0$ such that $(\beta_O \cap P(\bar{c}))\backslash(B(a) \cup W(a))$ contains a measurable set A' with

$\dfrac{\mu_{\bar{c}}(A')}{\mu_{\bar{c}}(\beta_O \cap P(\bar{c}))} > k'$. We then integrate across all $\beta_O \cap P(\bar{c})$, $\bar{c} \in \{a + p \in H\colon p \in S\}$, to conclude the proof. ■

Proof of Proposition 3. For any $j \in \mathbb{I}$ and $y \in F$, let β_y^j denote $\{x \in F\colon y_j \nsucc_j x_j\}$, which is nonempty since $y \in \beta_y^j$. Upper continuity implies that each β_y^j is closed. For any finite set of allocations, say $Y = \{y^1, ..., y^t\}$, the acyclicity of $\succ_i$ implies there is some $r \in \{1, ..., t\}$ such that $y_j^k \nsucc_j y_j^r$ for all $k \in \{1, ..., t\}$. Hence $y^r \in \beta_{y^k}^j$, $k \in \{1, ..., t\}$ and thus $y^r \in \beta_{y^k}^j \cap A$ for any $A \supset Y$. We conclude that for any nonempty $A \subset F$, the sets $\{\beta_y^j \cap A\}_{y \in A}$ enjoy the finite intersection property. Hence if $A \subset F$ is compact and nonempty, $\bigcap_{y \in A} (\beta_y^j \cap A) \neq \varnothing$. In particular, $A^0 = \{y \in F\colon w_i \nsucc_i y_i \text{ for all } i \in \mathbb{I}\}$ is compact and nonempty. We may therefore for $i = 1, ..., I$, inductively define the nonempty compact sets $A^i = \bigcap_{y \in A^{i-1}} (\beta_y^i \cap A^{i-1})$. Each allocation in A^I is Pareto optimal. To prove the second result, let A^0 instead denote $\{y \in F\colon y_i \succeq_i w_i \text{ for all } i \in \mathbb{I}\}$ and define the remaining A^i as before. ■

Proof of Proposition 6. Given Proposition 5, regarding the regular case it remains only to show that (I) for an open dense set of economies, PO_R is nonempty. For the nonregular case, it remains to show that for an open dense set of economies (II) any nonregular optimum is the limit of a sequence of regular optima, thus implying $PO_{NR} \subset \partial PO_R$ (where PO_{NR} denote the set of nonregular optima), and (III) the nonregular points have measure 0.

To show that I, II, and III hold for an open and dense set of economies, we (1) define a finite-dimensional set of parameters δ, which will establish property I, (2) define a product space of the δ's and the endogenous variables (x, n) and a map for each agent i such that, for any δ, is a submersion onto M_i (3) use this map and the transversality theorem to show that generically the M_i intersect transversally, (4) show that transversal intersection of the M_i implies property II, (5) add an additional transversality argument to show that for almost every allocation normal cones intersect transversally, which establishes property III.

We first extend each M_i to a boundaryless manifold. Redefine M_i to be a subset of $R^{LI} \times S$ rather than $R^L \times S$: $M_i = \{(x, n) \in R_+^{LI} \times S\colon n \in N_i(x_i)\}$. Now extend each M_i to a C^1

manifold with boundary M_i^{ex} so that the projection of M_i^{ex} onto its x coordinate, say R^{ex}, contains F and forms an open subset of the $L(I-1)$ affine subspace in R^{LI} that spans F.

(1) Given M_i^{ex} and $\delta = (\delta_1, \ldots \delta_I) \in (0, 1) \times \ldots \times (0, 1) \equiv \Delta$, define

$$M_i(\delta) = \{(x, n + b) \in R^{LI} \times S: b \in \beta(\delta_i, x) \text{ and } (x, n) \in M_i\},$$

where $\beta(\delta_i, x)$ is the ball in R^L with center 0 and radius $r(\delta_i, x) = \frac{\delta_i(2 - \operatorname{diam} N_i(x_i))}{2}$.[6] Also let $N_i(x, \delta)$ denote $\{n \in S: (x, n) \in M_i(\delta)\}$ and, given some $n \in S$, define $H_n = \{x_i \in R^L_+: x_i \cdot n \geq 0\}$. We set $B_{i,\delta}(x_i) = \operatorname{int}(B_i(x_i) \cap \bigcap_{n \in N_i(x_i, \delta)} H_n)$. (If $x_i \in B_i(x_i)$ and we wish to preserve reflexivity, we could instead set $B_{i,\delta}(x_i) = \operatorname{int}(B_i(x_i) \cap \bigcap_{n \in N_i(x_i, \delta)} H_n) \cup \{x_i\}$.) Since $N_i(x) \subset N_i(x, \delta)$, $B_{i,\delta}(x_i)$ will generate the normal cone $N_i(x, \delta)$, that is, if $\succeq_{i,\delta}$ is defined by

$$x_i \succeq_{i,\delta} y_i \text{ if and only if } x_i \in B_{i,\delta}(y_i)$$

then for any allocation x the normal cone of $\succeq_{i,\delta}$ at x_i will be given by $N_i(x, \delta)$. We have let the expansion of the N_i shrink to 0 as $\operatorname{diam} N_i(x_i)$ approaches 2 so that $\operatorname{diam} N_i(x, \delta) < 2$, thus ensuring that $N_i(x, \delta)$ remains the intersection of S and a convex cone, and hence that $B_{i,\delta}(x_i)$ is nonempty and therefore consistent with local nonsatiation. That $\succeq_{i,\delta}$ has smooth normals follows from that $\succeq_i$ has smooth normals and the fact that $r(\delta_i, x)$ is a C^1 function of x, while $\succeq_{i,\delta}$ is lower continuous since the "int" specification for $B_{i,\delta}(x_i)$ implies it is an open set (or at least that $B_{i,\delta}(x_i) \setminus \{x_i\}$ is open). Finally, because we only eliminate from and add no bundles to any B_i, and hence only eliminate ordered pairs from $\succeq_i$, $\succeq_{i,\delta}$ remains weakly transitive if $\succeq_i$ is. If $\delta^n \to \delta$ then each $\succeq_{i,\delta^n}$ converges to $\succeq_{i,\delta}$. Observe that if x is Pareto optimal for the original economy with M_i, $i \in \mathbb{I}$, then, for any $\delta \in \Delta$, x is a regular Pareto optimum for $M_i(\delta)$, $i \in \mathbb{I}$. Thus the set of economies for which PO_R is nonempty forms a dense (and, self-evidently, open) set: property I is satisfied.

(2) Let F^{ex} denote the $L(I-1)$-manifold formed by the intersection of R^{ex} and the affine subspace spanned by F. Since, for any x and δ, $\partial N_i(x, \delta)$ is compact and boundaryless the ε-

[6] For any $A \subset R^m$, $\operatorname{diam} A = \sup\{\|x - y\|: x \in A, y \in A\}$.

neighborhood theorem (see, e.g., Guillemin and Pollack (1974), 2.3) implies there exists a ε neighborhood of $\partial N_i(x, \delta)$ in, and open relative to, S, and a C^1 submersion from this neighborhood to $\partial N_i(x, \delta)$ that is the identity on $\partial N_i(x)$; this function can be chosen to take each n to the $\hat{n} \in \partial N_i(x, \delta)$ that minimizes $\|n - \hat{n}\|$. By adjusting the proof of the neighborhood theorem slightly, one may show that there is a set $\hat{FS} \times \hat{\Delta}$ in, and open relative to, $(F^{ex} \times S) \times \Delta$, and a C^1 map G_i: $\hat{FS} \times \hat{\Delta} \rightarrow \hat{FS}$ such that (i) $\mathrm{proj}_{F^{ex}} \hat{FS} \supset F$, (ii) $\hat{\Delta} \subset R_+^I$ is an open rectangle with $0 \in \mathrm{cl}\,\hat{\Delta}$, (iii) if $(x, n) \in \partial M_i(\delta)$ and $\delta \in \hat{\Delta}$, then $(x, n) \in \hat{FS}$, (iv) G_i maps $(x, n, \delta) \in \hat{FS} \times \hat{\Delta}$ to $(x, \hat{n})$, where $\hat{n} \in \partial N_i(x, \delta)$ minimizes $\|n - \hat{n}\|$, and (v) for $\delta \in \hat{\Delta}$, g_i^δ: $\hat{FS} \rightarrow \hat{FS}$ defined by $g_i^\delta(x, n) = G_i(x, n, \delta)$ is a submersion onto $\partial M_i(\delta)$. (Here and subsequently ∂M_i and $\mathrm{int} M_i$ will refer to the boundary and interior of the manifold M_i.) Property (iv) is not essential, but it simplifies the calculation of a derivative in (3).

(3) To show, for any C^1 submanifold P of $\hat{FS}$, that $G_i \pitchfork P$, it is sufficient for $\dim(\mathrm{Image}\, DG_i(x, n, \delta))$ to equal $\dim \hat{FS} = L(I - 1) + L - 1$ for any $(x, n, \delta) \in \hat{FS} \times \hat{\Delta}$. Since for $\delta \in \hat{\Delta}$, g_i^δ is a submersion onto $\partial M_i(\delta)$, and $\partial M_i(\delta)$ has dimension equal to $\dim(\hat{FS}) - 1$, we have $\dim(\mathrm{Image}\, DG_i(x, n, \delta)) \geq \dim(\hat{FS}) - 1$. Moreover, $\dim(\mathrm{Image}\, DG_i(x, n, \delta)) = \dim(\hat{FS})$ if, for any $\delta \in \hat{\Delta}$ and $(x, n) \in \partial M_i(\delta)$, $\mathrm{Image}(DG_i(x, n, \delta))$ contains some direction not in $T_{x,n} \partial M_i(\delta)$. ($T_y A$ will denote the tangent bundle of a manifold A at y.) For $m \in T_{\hat{FS}}$ given by $(0, n' \neq 0)$, where $n' \cdot n = 0$ for all $n \in \partial N_i(x, \delta)$, we have $m \perp T_{x,n} \partial M_i(\delta)$. Since $D_{\delta_i} G_i((x, n), \delta) = \dfrac{r(\delta_i, x)}{\|m\|} m$, $D_{\delta_i} G_i((x, n), \delta) m \neq 0$ and so m may serve as the additional direction.

For the submanifold $\partial M_j(\delta)$ of $\hat{FS}$, $j \neq i$, the transversality theorem implies that the $\delta \in \hat{\Delta}$ such that $g_i^\delta \pitchfork \partial M_j(\delta)$ form a set $\Delta_{ij} \subset \hat{\Delta}$ whose complement in $\hat{\Delta}$ has 0 measure. Since g_i^δ is a submersion onto $\partial M_i(\delta)$, Image $Dg_i^\delta(x, n)$ coincides with $\mathrm{Image}\, D\imath(M_i(\delta))$, where $\imath(M_i(\delta))$: $\partial M_i(\delta) \hookrightarrow \hat{FS}$ is the inclusion map of $\partial M_i(\delta)$ and so $\partial M_i(\delta) \pitchfork \partial M_j(\delta)$ for $\delta \in \Delta_{ij}$. Moreover, since F is compact, the set $\overline{\Delta}_{ij} \subset \hat{\Delta}$ such that $\partial M_i(\delta) \pitchfork \partial M_j(\delta)$ on F in addition to (i) having a 0-measure complement and (ii) containing 0 in its closure is also (iii) open. Call any subset of $\hat{\Delta}$ with these three properties *generic*. Apply the same logic to any pair (k, l) of agents and take the intersection of the resulting $\overline{\Delta}_{kl}$, thus arriving at a generic set Δ^2. Since for any $\delta \in \Delta^2$ and any $i, j, l \in \mathbb{I}$, $\partial M_j(\delta) \pitchfork \partial M_l(\delta)$, $\partial M_j(\delta) \cap \partial M_l(\delta)$ is a C^1 manifold and hence there is a generic set $\overline{\Delta}_{i,j,l}$ such that

$$(T) \qquad \partial M_i(\delta) \pitchfork (\partial M_j(\delta) \cap \partial M_l(\delta))$$

holds for any $\delta \in \overline{\Delta}_{i,j,l}$. Consequently there is also a generic set Δ^3 such that T holds for any triple in $\mathbb{I}$ and $\delta \in \Delta^3$. Proceeding by induction we conclude there is a generic set Δ^I such that for any $\delta \in \Delta^I$ and any agent i and $I_i \subset \mathbb{I} \setminus \{i\}$, $\partial M_i(\delta) \pitchfork \bigcap_{j \in I_i} \partial M_j(\delta)$.

(4) For any $\delta \in \Delta^I$ and $(x, n) \in \bigcap_{i \in \mathbb{I}} M_i(\delta)$ (that is, an optimal x supported by n), consider the $\hat{I} \subset \mathbb{I}$ defined by $i \in \hat{I}$ if and only if $(x, n) \in \partial N_i(\delta)$ (that is, the agents for whom n in on the boundary of their normal cones). Relabel agents so that $\hat{I} = \{1, ..., t\}$. Since

$$T_{x,n}\partial M_1(\delta) + T_{x,n}(\textstyle\bigcap_{j=2}^t \partial M_j(\delta)) = R^{\dim F\hat{S}},$$

there must be $(x^1, n^1) \in (\text{int}\, M_1(\delta)) \cap \bigcap_{j=2}^t \partial M_j(\delta)$ arbitrarily near (x, n). Similarly since

$$T_{x,n}\partial M_2(\delta) + T_{x,n}(\textstyle\bigcap_{j=3}^t \partial M_j(\delta)) = R^{\dim F\hat{S}},$$

there must exist $(x^2, n^2) \in (\text{int}\, M_2(\delta)) \cap \bigcap_{j=3}^t \partial M_j(\delta)$ arbitrarily near (x^1, n^1) and hence still in $\text{int}\, M_i(\delta)$. Proceeding by induction we conclude there is a $(x^t, n^t) \in \bigcap_{j=1}^t \text{int}\, M_j(\delta)$ that may be chosen to be arbitrarily near (x, n). Since $(x^t, n^t) \in \bigcap_{j=1}^t \text{int}\, M_j(\delta)$ implies $n^t \in \bigcap_{j=1}^t \text{int}_S\, N_j(x^t, \delta)$, we conclude that for any $\delta \in \Delta^I$ and any interior optimum x there is a sequence of regular optima that converges to x. For a boundary x in contrast, it may be that any $x^t \notin F$. To cover the nonregular boundary optima, we can apply (with no alterations) the logic from (2) onwards to an arbitrary coordinate subspace. Specifically, for each good k let $I(k)$ denote an arbitrary strict subset of $\mathbb{I}$ (indicating the agents who do not consume k), let $\overline{R} = \{x \in R_+^{LI}: i \in I(k) \text{ implies } x_i(k) = 0\}$, and let $\overline{F} = F^{ex} \cap \overline{R}$. Letting $\overline{F}$ take the place of F^{ex}, we conclude, for δ in a generic set, that for any nonregular boundary optimum in $\overline{F} \cap F$ we may find a $x^t \in \overline{F} \cap F$ arbitrarily near x such that $\bigcap_{j=1}^t \text{int}_S\, N_j(x^t, \delta) \neq \varnothing$. While x^t need not be optimal (since it is a boundary point), any interior point sufficiently near x^t must be. Thus the property $PO_{NR} \subset \partial PO_R$ is dense. Since $PO_{NR} \subset \partial PO_R$ follows from our manifolds having transversal intersection, the openness of the property $PO_{NR} \subset \partial PO_R$ follows as usual from the compactness of F.

(5) Next we show that PO_{NR} has measure 0 for any $\delta \in \Delta^I$. Fix some $\delta \in \Delta^I$. For any $x \in F$ and $i \in \mathbb{I}$, let $\tilde{F}_x$ and $\tilde{S}_x$, with $\tilde{F}_x \times \tilde{S}_x \subset F\hat{S}$ and open relative to F_{ex} and S respectively, be such that if $(y, n) \in \partial M_i(\delta)$ and $y \in \tilde{F}_x$ then $n \in \tilde{S}_x$. Since $\{\tilde{F}_x\}_{x \in F}$ covers the compact set F, we can restrict ourselves to some finite selection from $\{\tilde{F}_x\}_{x \in F}$ that covers F. Since $\delta \in \Delta^I$, we know that $g_i^\delta \pitchfork \bigcap_{j \in I_i} \partial M_j(\delta)$ for any i and $I_i \subset \mathbb{I} \setminus \{i\}$. Hence by the transversality theorem the function $h_i^{\delta, y}\colon \tilde{S}_x \to F\hat{S}$ defined by $h_i^{\delta, y}(n) = g_i^\delta(x, n)$ satisfies $h_i^{\delta, y} \pitchfork \bigcap_{j \in I_i} \partial M_j(\delta)$ for a.e. $y \in \tilde{F}_x$. For any of these y and any n such that $(y, n) \in \partial M_i(\delta) \cap \bigcap_{j \in I_i} \partial M_j(\delta)$, Image $Dh_i^{\delta, y}(n)$ has dimension equal to $\dim S - 1$ and consists only of directions $(0, \hat{n})$ where $\hat{n} \in T_n \partial N_i(y, \delta)$. For $n' \in T_n S$ such that $(0, n') \perp$ Image $Dh_i^{\delta, y}(n)$, it must be (given $h_i^{\delta, y} \pitchfork \bigcap_{j \in I_i} \partial M_j(\delta)$) that $(0, n') \in T_{(y,n)}(\bigcap_{j \in I_i} \partial M_j(\delta))$. Hence $\partial N_i(y, \delta) \pitchfork \bigcap_{j \in I_i} \partial N_j(y, \delta)$. Given i and $\tilde{F}_x$, we can specify such a set of y in $\tilde{F}_x$, each with null complement in $\tilde{F}_x$, for any of the finite number of $I_i \subset \mathbb{I} \setminus \{i\}$. Letting the finite selection from $\{\tilde{F}_x\}_{x \in F}$ vary and then letting i vary and taking the intersection of the resulting finite number of sets, we conclude that any y outside of a null set of allocations has $\partial N_i(y, \delta) \pitchfork \bigcap_{j \in I_i} \partial N_j(y, \delta)$ for all i and I_i. Hence any such y that is an optimum is a regular optimum. As in the previous paragraph, therefore, for any of these y that are optimal, $\bigcap_{i \in \mathbb{I}} \operatorname{int}_S N_i(y, \delta) \neq \varnothing$. Openness of the property of PO_{NR} having measure 0 follows again from the compactness of F. ■

Proof of Proposition 7. For $z \in R_+^{LI}$, let z_2 denote the projection of z onto the coordinates (i, k) for which agent i displays status quo bias for good k at z, and let z_1 denote the projection of z onto the remaining coordinates. Since $\succeq_i(z) = \succeq_i(y)$ whenever $z_2 = y_2$, we write $\succeq_i(z_2)$ instead of $\succeq_i(z)$. Let $\bar{x}$ denote the interior regular optimum given in the proposition and let $\bar{p} = (\bar{p}_1, \bar{p}_2)$ have the property, guaranteed by the regularity of $\bar{x}$, that if i displays status quo bias for k at $\bar{x}_i$ then $\bar{p} \in \operatorname{int}_{S_k(p)}(N_i(\bar{x}_i) \cap S_k(p))$. 'No isolated communities' and well-behaved conditional preferences imply, for any z_2, that $PO(z_2) = \{z_1\colon z_1$ is Pareto optimal for $\succeq_i(z_2)$, $i \in \mathbb{I}\}$ has dimension $I - 1$. Hence, given the continuous normals assumption, it is sufficient to show, for any $x_2^n \to \bar{x}_2$ and any $x_1^n \to \bar{x}_1$ with $x_1^n \in PO(x_2^n)$, that $x^n = (x_1^n, x_2^n)$ is Pareto optimal for all n sufficiently large. Since $x_1^n \in PO(x_2^n)$, the continuous normals assumption implies that there exists $p_1^n \to \bar{p}_1$ such that, for each agent i, $p_1^n \cdot (y_{1i} - x_{1i}^n) \geq 0$ for all $y_{1i} \succeq_i (x_2^n)\ x_{1i}^n$. To show that, for large n, x^n is Pareto optimal and supported by $(p_1^n, \bar{p}_2)$, let us suppose to the contrary that there is a subsequence x^n such that for each n there exists a i and y_i^n with $y_i^n \succeq_i x_i^n$ and $(p_1^n, \bar{p}_2) \cdot (y_i^n - x_i^n) < 0$. Taking a further subsequence if necessary we suppose that this i remains the same for all n. Our selection of p_1^n implies that $y_{i2}^n \neq x_{i2}^n$.

Let $\hat{z}^n \in S$ denote $\frac{1}{\|y_i^n - x_i^n\|}(y_i^n - x_i^n)$ and let $N_i^\circ(x_i)$ denote the normalized polar cone of $N_i(x_i)$: $N_i^\circ(x_i) \equiv \{z \in S\colon z \cdot p \geq 0$ for all $p \in N_i(x_i)\}$. Then $w \in N_i^\circ(x_i)$ if and only if there are sequences $\lambda^t > 0$ and y_i^t such that $y_i^t \in B_i(x_i)$ and $\lambda^t(y_i^t - x_i) \to w$. In particular each $\hat{z}^n \in N_i^\circ(x_i^n)$. Since, for any x_i, the (normalized) polar cone of $N_i^\circ(x_i)$ is $N_i(x_i)$, $\hat{z}^n \cdot p \geq 0$ for all $p \in N_i(x_i^n)$. The continuous normals assumption therefore implies that any accumulation point of $\hat{z}^n$, say $\bar{z}$, satisfies $\bar{z} \cdot p \geq 0$ for all $p \in N_i(\bar{x}_i)$. Thus $\bar{z} \in N_i^\circ(\bar{x}_i)$. On the other hand, since by assumption $(p_1^n, \bar{p}_2) \cdot (y_i^n - x_i^n) < 0$ for each n, $\bar{p} \cdot \bar{z} \leq 0$. Since $y_{i2}^n \neq x_{i2}^n$ and therefore $\hat{z}_2(k) \neq 0$ for some coordinate k, for any $\varepsilon > 0$ we can find $p' \in S_k(\bar{p})$ with $\|p' - \bar{p}\| < \varepsilon$ and $p' \cdot \bar{z} < 0$, and since i displays status quo bias for k, $p' \in N_i(\bar{x}_i)$ when p' is sufficiently near $\bar{p}$. But then $p' \cdot (y_i^t - \bar{x}_i) < 0$ for some $y_i^t \in B_i(\bar{x}_i)$, a contradiction. ■

Proof of Proposition 9. Suppose x Pareto dominates y: $x_i \succeq_i y_i$ for all $i \in \mathbb{I}$ and $x_j \succ_i y_j$ for some $j \in \mathbb{I}$. Then for the $(v_i[\![1]\!], ..., v_i[\![L]\!]) \in V_i$, $i \in \mathbb{I}$, given by Pareto compatibility, $\Sigma_{k\in\mathcal{L}} v_i[\![k]\!](x_i) \geq \Sigma_{k\in\mathcal{L}} v_i[\![k]\!](y_i)$ for all i. If $\Sigma_{k\in\mathcal{L}} v_j[\![k]\!](x_j) = \Sigma_{k\in\mathcal{L}} v_j[\![k]\!](y_j)$ then x_j is not a global minimum and hence not a local minimum of $v_j[\![k]\!]$, and so there is a sequence $z_{jt} \to x_j$ such that $\Sigma_{k\in\mathcal{L}} v_j[\![k]\!](z_{jt}) < \Sigma_{k\in\mathcal{L}} v_j[\![k]\!](x_j) = \Sigma_{k\in\mathcal{L}} v_j[\![k]\!](y_j)$, violating lower continuity. Thus $\Sigma_{k\in\mathcal{L}} v_j[\![k]\!](x_j) > \Sigma_{k\in\mathcal{L}} v_j[\![k]\!](y_j)$ and hence $\Sigma_{i\in\mathbb{I}} \Sigma_{k\in\mathcal{L}} v_i[\![k]\!](x_i) > \Sigma_{i\in\mathbb{I}} \Sigma_{k\in\mathcal{L}} v_i[\![k]\!](y_i)$. Then y cannot be a utilitarian optimum: if it were, separability implies $\Sigma_{i\in\mathbb{I}} v_i[\![k]\!](y_i) \geq$

$\Sigma_{i \in \mathbb{I}} v_i [\![k]\!](x_i)$ and hence $\Sigma_{k \in \mathscr{L}} \Sigma_{i \in \mathbb{I}} v_i [\![k]\!](y_i) \geq \Sigma_{k \in \mathscr{L}} \Sigma_{i \in \mathbb{I}} v_i [\![k]\!](x_i)$. ■

Addendum to proof of Proposition 9. If $V_i = V_i [\![1]\!] \times ... \times V_i [\![L]\!]$ and $v_i [\![k]\!] \in V_i [\![k]\!]$ is concave for each i and k, then any utilitarian optimum y maximizes $\Sigma_{k \in \mathscr{L}} \delta_k \Sigma_{i \in \mathbb{I}} \bar{v}_i [\![k]\!]$ for some $\delta = (\delta_1, ..., \delta_I) \geq 0$ and $\bar{v}_i \in V_i$. Hence if $\delta \gg 0$ then we may apply the above proof using $(\delta_i \bar{v}_i [\![1]\!], ..., \delta_i \bar{v}_i [\![L]\!]) \in V_i$, $i \in \mathbb{I}$, to represent i's preferences and (assuming no local minima) to conclude that $\Sigma_{i \in \mathbb{I}} \Sigma_{k \in \mathscr{L}} \delta_i \bar{v}_i [\![k]\!](x_i) > \Sigma_{i \in \mathbb{I}} \Sigma_{k \in \mathscr{L}} \delta_i \bar{v}_i [\![k]\!](y_i)$ if x Pareto dominates y, contradicting the maximality of y. Hence, modulo the technical provisos, $V_i = V_i [\![1]\!] \times ... \times V_i [\![L]\!]$ and concavity imply that a utilitarian optimum is Pareto optimal.

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